Playsheets 19 (front) and 20 (back)
More About Graphs

MATH 130
Thursday, April 30, 2009

Directions: Work together on each problem; do not delegate different problems to different people. Submit one **neatly written** write-up per group. Remember to use complete sentences as appropriate and to **show your work**.

1. (a) Draw K_4 and K_5 and determine their chromatic numbers.

   (b) Based on your work in (a), what is the chromatic number of K_n? Why?

2. A **cycle** graph is a graph that is just a cycle; a cycle with n vertices is denoted C_n. For example, C_3 is a triangle, C_4 is a square, etc.

   (a) Draw C_3, C_4, C_5, and C_6.

   (b) Determine the chromatic number of each cycle in (a). (You may work on the graphs you drew above.)

   (c) Generalize your results in part (b): what is the chromatic number of C_n? Why?

3. A **bipartite** graph is a graph that can be drawn with the vertices in two rows in such a way that the only edges that appear join vertices from different rows. (Not all such edges have to appear.)

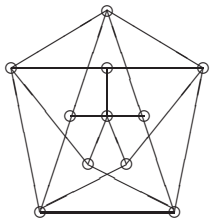
   (a) Draw a bipartite graph with three vertices in the top row and four in the bottom row.

   (b) A **complete bipartite** graph is a bipartite graph in which every edge of the top row is joined to every edge of the bottom row. If the rows have m and n vertices, the complete bipartite graph is denoted $K_{m,n}$. Draw $K_{3,3}$ and $K_{2,4}$.

   (c) Determine the chromatic number of each of your bipartite graphs from (a) and (b). (You may work on the graphs you drew above.)

   (d) Use your work above to conjecture what $\chi(K_{m,n})$ is. Then argue that your conjecture is correct.

4. Find $\chi(G)$ for the graph G below. Explain why you are correct.



5. Explain how the Four-Color Theorem can be used to show that K_5 is not planar.

6. Explain why the Four-Color Theorem *cannot* be used to show that $K_{3,3}$ is not planar.

7. Find an embedding of K_6 on the torus. Use the "rectangle" version of the torus.

8. Find an embedding of the Petersen graph that has no crossings on the torus.

9. The map below is to be colored with red, blue, yellow, and green. With the colors as shown below, show that country A must be colored red. What can you say about the color of country B? [Source: Wilson and Watkins.]