Playsheet 14
My Imaginary Friend

MATH 130-02
Thursday, April 2, 2009

Directions: Groups should consist of three or four people. Work together on each problem; do not delegate different problems to different people. Submit one **neatly written** write-up per group. Remember to use complete sentences as appropriate and explain your reasoning. That is, **show your work!**

1. Recall the number i having the property that $i^2 = -1$. This number is called the **imaginary unit**.
   (a) Compute i^3, i^4, i^5, i^6, i^7, and i^8.

   (b) What is i^{75}?

   (c) What is i^n in general?

2. The **complex numbers**, denoted $\mathbb{C}$, are made up of all numbers of the form $a + bi$, where a and b are real numbers. ($7 + 3i, \pi - \frac{2}{5}i$, etc.)
   (a) Addition is defined by $(a + bi) + (c + di) = (a + c) + (b + d)i$. Compute each sum below.
      i. $(4 + 3i) + (7 + 5i)$

      ii. $(6 - 3i) - (8 + 5i)$

   (b) Multiplication is performed via "FOIL" à la polynomials. Compute each product below. Remember that $i^2 = -1$!
      i. $(4 + 3i)(7 + 5i)$

      ii. $(6 - 3i)(8 + 5i)$

      iii. $(a + bi)(a + bi)$

(OVER)

3. Complex numbers also have a *geometry* associated with them. A complex number $a + bi$ gets graphed at the point (a, b) in a standard xy-plane. The x-axis is referred to as the **real axis** and the y-axis is the **imaginary axis**. Graph each complex number on the graph paper provided. (Graph them all on the same set of axes and label each point by letter.)

   (a) $4 + 3i$

   (b) $7 + 5i$

   (c) $(4 + 3i) + (7 + 5i)$

   (d) $(4 + 3i)(7 + 5i)$

   (e) $2 + i$

   (f) $(2 + i)^2$

   (g) $4 - i$

   (h) $(4 - i)^2$

4. What happens to the number $1 + i$ when it is repeatedly squared? Graph the results on your graph paper.

5. What happens to the number $0.6 + 0.8i$ when it is repeatedly squared? Graph the results on your graph paper.

6. What happens to the number $0.6+0.8i$ when the process of squaring and then subtracting 1 is repeatedly applied? Graph the results on your graph paper.