Playsheet 15
Fractal Dimension

MATH 130-02
4/7/2009

Directions: Groups should consist of three or four people. Work together on each problem; do not delegate different problems to different people. Submit one **neatly written** write-up per group. Remember to use complete sentences as appropriate and explain your reasoning. That is, **show your work!**

1. Recall that two geometrical objects are **similar** if one is just a scaled version of the other. (One could be the image of the other on an overhead projector, for example; a photograph of a flat object would be similar to the original object, too.)

   (a) Draw two similar triangles.

   (b) Draw a line segment 1 centimeter long (approximately). How many line segments identical to that one would it take to create a new line segment similar to the original (but twice as long)? Draw the new, longer line segment, too.

   (c) Draw a square that measures one centimeter on a side. How many squares identical to that one would it take to create a new square similar to the original (but twice as wide)? Draw the new, larger square, too. Show subdivisions indicating where you have placed copies of the original square to create the new one.

   (d) Draw a cube that measures one centimeter on a side. How many cubes identical to that one would it take to create a new cube similar to the original (but twice as wide)? Draw the new, larger cube, too. Show subdivisions indicating where you have placed copies of the original cube to create the new one.

   (e) For each of the objects above, how many would it take if you were to scale by a factor of 3 instead of 2?

2. Write your answers to (a), (b), and (c) above as powers of 2. That is, write each one as 2^d for some d. For part (d), express your answers as powers of 3. What values of d did you find in each case? Did your values of d match for doubling and tripling? The number d is called the **dimension** of the original object.

3. Now we compute the dimension of the Koch snowflake curve. (Gulp!)

   (a) Draw a portion of one "side" of the Koch curve by starting with a one-centimeter line segment building triangles on middle thirds (as before). Do three iterations of this.

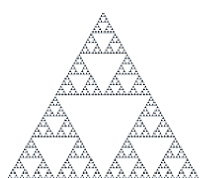
(OVER)

(b) Now draw a copy of your curve in (a) that is three times as wide, then perform one more iteration on this new curve. (Remember that the real curve has had infinitely many iterations performed! We're just looking at a part we can actually draw.)

(c) How many copies of the curve in (a) could you put together to create the curve in (b)? Call this number c.

(d) Following our work in number 1, we want to write the number of copies as a power of 3 (our scale factor): $c = 3^d$. The number d that works is $\ln(c)/\ln(3)$. Use your calculator to compute this number. It is the **fractal dimension** of the Koch snowflake curve.

4. Now consider the Sierpinski triangle.



(a) How many copies of the triangle would it take to make another (larger) copy of it? Call this number c. How wide would the larger copy be compared to the original? Call this number m (for magnification).

(b) We need the number d such that $c = m^d$. You can find it on your calculator by computing $\ln(c)/\ln(m)$.

5. One more: Find the fractal dimension of the Sierpinski carpet: