

Rules of Differentiation

We have the following rules of differentiation. In all cases, f and g are differentiable functions.

MATH 139–02

| Name | Conditions | Rule | Example 1 | Example 2 |
|------------------------|---------------------------|---|---|--|
| Constant Rule | c is a constant | $\frac{d}{dx}(c) = 0$ | $\frac{d}{dx}(5) = 0$ | $\frac{d}{dx}(-4, 856, 324) = 0$ |
| Power Rule | n is a constant | $\frac{d}{dx}(x^n) = nx^{n-1}$ | $\frac{d}{dx}(x^6) = 6x^5$ | $\frac{d}{dx}x^{-1} = -x^{-2}$ |
| Constant Multiple Rule | c is a constant | $\frac{d}{dx}(cf(x)) = cf'(x)$ | $\frac{d}{dx}(3x^4) = 3 \cdot 4x^3$ | $\frac{d}{dx}(-5x^{3/4}) = -5 \cdot \frac{3}{4}x^{-1/4}$ |
| Natural Logarithm | | $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$ | | |
| Natural Exponential | | $\frac{d}{dx}(e^x) = e^x$ | | |
| Exponential functions | $a > 0$, a is constant | $\frac{d}{dx}(a^x) = a^x \ln(a)$ | $\frac{d}{dx}(3^x) = 3^x \ln(3)$ | $\frac{d}{dx}[3(1.05)^x]$ $= 3 \cdot (1.05)^x \ln(1.05)$ |
| Sum/Difference Rules | | $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$ | $\frac{d}{dx}(e^x + x^4) = e^x + 4x^3$ | $\frac{d}{dx}(3x^3 + 5x^2 - 1)$ $= 3 \cdot 3x^2 + 5 \cdot 2x$ |
| Product Rule | | $\frac{d}{dx}(f(x) \cdot g(x))$ $= f'(x)g(x) + g'(x)f(x)$ | $\frac{d}{dx}(x^2(x^3 - 2))$ $= 2x(x^3 - 2) + 3x^2(x^2)$ | $\frac{d}{dx}(x^2 \ln(x))$ $= 2x \ln(x) + \frac{1}{x}(x^2)$ |
| Quotient Rule | $g(x) \neq 0$ | $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right)$ $= \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$ | $\frac{d}{dx} \left(\frac{x}{x^2 + 1} \right)$ $= \frac{1(x^2 + 1) - 2x(x)}{(x^2 + 1)^2}$ | $\frac{d}{dx} \left(\frac{e^x}{x} \right)$ $= \frac{e^x(x) - 1(e^x)}{x^2}$ |
| Chain Rule | | $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$ | $\frac{d}{dx}(e^{x^2+3x}) = e^{x^2+3x}(2x + 3)$ | $\frac{d}{dx}((x^3 + 2x)^4)$ $= 4(x^3 + 2x)^3(3x^2 + 2)$ |