Solutions to Quiz 4

MATH 139-01 and -02
Monday, September 15, 2003

Be sure to **show your work**. Unsupported answers receive no credit.

1. If the quantity of a substance decreases by 4% in 10 hours, find its half-life.

   Solution: We have a quantity of $Q(t) = Q_0(0.96)^{t/10}$ after t hours, and we need to find t such that $Q(t) = \frac{1}{2}Q_0$. Solve:

$$\begin{aligned} Q_0(0.96)^{t/10} &= \frac{1}{2}Q_0 \\ (0.96)^{t/10} &= 0.5 \\ \ln(0.96^{t/10}) &= \ln(0.5) \\ \frac{t}{10}\ln(0.96) &= \ln(0.5) \\ t &= \frac{10\ln(0.5)}{\ln(0.96)} \\ t &\approx 169.8. \end{aligned}$$

   Thus, the half-life is about 169.8 hours.

2. A radioactive substance has a half-life of 8 years. If 200 grams are present initially, how much remains at the end of 12 years?

   Solution: The amount after t years is $Q(t) = 200(0.5)^{t/8}$. Thus, after 12 years the amount is $Q(12) = 200(0.5)^{12/8} \approx 70.7$ grams. To find out when 10% remains, we must solve:

$$\begin{aligned} 200(0.5)^{t/8} &= 20 \\ (0.5)^{t/8} &= 0.1 \\ \frac{t}{8}\ln(0.5) &= \ln(0.1) \\ t &= \frac{8\ln(0.1)}{\ln(0.5)} \\ t &\approx 26.6. \end{aligned}$$

   It will take about 26.6 years to get the amount down to 10%.