Solutions to Quiz 5

MATH 139-01 and -02
Tuesday, September 16, 2003

Be sure to **show your work**. Unsupported answers receive no credit.

1. Recall that if P dollars are invested at an annual interest rate of r for t years, compounding n times per year, then the balance after t years is

$$B(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

   (a) If $P = \$250$, $r = 0.08$, and interest is compounded monthly, what is the balance after 40 years?

   Solution: $B(40) = 250\left(1 + \dfrac{.08}{12}\right)^{12\cdot 40} \approx 6068.35.$

   (b) If $P = \$250$, $r = 0.08$, and interest is compounded daily, what is the balance after 40 years?

   Solution: $B(40) = 250\left(1 + \dfrac{.08}{365}\right)^{365\cdot 40} \approx 6130.98.$

   (c) How much more interest is earned if it is compounded daily rather than monthly?

   Solution: $6130.98 - 6068.35 = 62.63$. You'd earn about \$62.63 over the course of 40 years. That doesn't sound like a lot, but notice that it's about an additional quarter of your original investment!

   (d) How long must one wait before the balance reaches \$10,000?

   Solution: I will solve with both daily and monthly compounding:

   For monthly compounding:

$$\begin{aligned}
250\left(1 + \frac{0.08}{12}\right)^{12t} &= 10000 \\
\left(1 + \frac{0.08}{12}\right)^{12t} &= (40) \\
12t \ln\left(1 + \frac{0.08}{12}\right) &= \ln 40 \\
t &= \frac{\ln(40)}{12\left(1 + \dfrac{0.08}{12}\right)} \\
t &\approx 46.26
\end{aligned}$$

   years. That's not much longer than it took to get to the \$6000 mark!

$$
\begin{aligned}
250\left(1+\frac{0.08}{365}\right)^{365t} &= 10000 \\
\left(1+\frac{0.08}{365}\right)^{365t} &= 40 \\
365t\ln\left(1+\frac{0.08}{365}\right) &= \ln(40) \\
t &= \frac{\ln(40)}{365\left(1+\frac{0.08}{365}\right)} \\
t &\approx 46.11
\end{aligned}
$$

years. It is only a couple of months less than monthly compounding.