Solutions to Quiz 10

MATH 139-02
Thursday, March 18, 2004

1. Let $f(x) = x^3 - 3x^2 - 9x + 15$.

   (a) Find the local extrema of $f(x)$. Give the x- and y-coordinates of each extremum you find. You may use your calculator to check your work, but you must use calculus to find the extrema!

   Solution: $f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x - 3)(x + 1)$. Thus, f' is zero if $x = -1$ or $x = 3$, so these are the candidates for local extrema. We have $f(-1) = 20$ and $f(3) = -12$, and a quick check of the graph shows that $(-1, 20)$ is a local maximum and $(3, -12)$ is a local minimum.

   (b) Use the first derivative test to determine for each extremum whether it is a maximum or a minimum.

   Solution: Since $f'(-2) = 3(-2 - 3)(-2 + 1) > 0$ and $f'(0) = -9 < 0$, $(-1, 20)$ must be a local maximum. SInce $f'(0) < 0$ and $f'(4) = 3(4 - 3)(4 + 1) > 0$, $(3, -12)$ must be a local minimum.

   (c) Now use the second derivative test to verify your results in part (b).

   Solution: $f''(x) = 6x - 6$. Since $f''(-1) = -12 < 0$, the graph is concave down here and we have a local maximum at $(-1, 20)$. Since $f''(3) = 12 > 0$, the graph is concave up here and we have a local minimum at $(3, -12)$.

2. Sketch the graph of a function that has a critical point at $x = 0$ but does **not** have a local extremum at $x = 0$.

   Solution: The graph of $y = x^3$ is such a graph.