# Solutions to Quiz 11

**MATH 139-02**
**Thursday, March 18, 2004**

1. Find all inflection points of $f(x) = x^3 - 12x^2 + 3x + 2$.

   **Solution:** $f'(x) = 3x^2 - 24x + 3$ and $f''(x) = 6x - 24$, so the only candidate is $x = 4$. Since $f''(0) < 0$ and $f''(5) = 6 > 0$, the concavity changes at $x =$. $f(4) = -114$, so $(4, -114)$ is an inflection point.

2. Find all inflection points of $f(x) = xe^x$.

   **Solution:** $f'(x) = e^x + xe^x = (x+1)e^x$. $f''(x) = e^x + (x+1)e^x = (x+2)e^x$. This is only zero if $x = -2$, so that is the only candidate for an inflection point. Since $f''(-3) = -e^{-3} < 0$ and $f''(0) = 2 > 0$, the concavity does change at $x = -2$. $f(-2) = -2e^{-2}$, so $(-2, -2e^{-2})$ is an inflection point.

3. Find all inflection points of $f(x) = x^4 - 2x^3 - 12x^2 + 7x - 3$.

   **Solution:** $f'(x) = 4x^3 - 6x^2 - 24x + 7$ and $f''(x) = 12x^2 - 12x - 24 = 12(x^2 - x - 2) = 12(x-2)(x+1)$. This is only zero if $x = 2$ or $x = -1$, so those are the only candidates for inflection points. Test: $f''(-2) = 12(-2-2)(-2+1) > 0$, $f''(0) = -24 < 0$, and $f''(3) = 12(3-2)(3+1) > 0$, so the concavity changes at both points. $f(2) = -33$ and $f(-1) = 21$, so both $(2, -33)$ and $(-1, 21)$ are inflection points.