Solutions to Quiz 15

MATH 139-02
Tuesday, April 20, 2004

1. Find the average value of $f(x) = x^2 + 1$ on $[0, 2]$.

   Solution: The average value is $\dfrac{1}{2-0}\displaystyle\int_0^2 (x^2+1)\,dx = \frac{1}{2}\left(\frac{1}{3}x^3 + x\right)\Big|_0^2$
   $= \dfrac{1}{2}\left[\left(\dfrac{1}{3}(2)^3 + 2\right) - \left(\dfrac{1}{3}(0^3) + 0\right)\right] = \dfrac{7}{3}$.

2. A particle had a velocity of $v(t) = \dfrac{t^4}{16}$ meters per second after t seconds had elapsed. What was the average speed of the particle during the first four seconds?

   Solution: The average speed is the average value of this function, which is
   $\dfrac{1}{4}\displaystyle\int_0^4 \frac{t^4}{16}\,dt = \frac{t^5}{320}\Big|_0^4 = \frac{4^5}{320} = \frac{16}{5} = 3.2$ meters per second.

3. On the graph below, sketch a rectangle whose height is equal to the average value of the function graphed on the interval $[-2, 2]$. What is the geometrical significance of this rectangle? (That is, what does it have to do with the function graphed?)