

# MTH 139-02

Exam 1

Tuesday, September 23, 2003

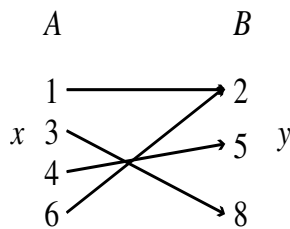
Name: \_\_\_\_\_

Remember to **show your work**. Unsupported solutions will receive no credit.

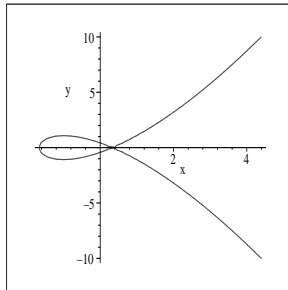
Calculus is the most powerful weapon of thought yet devised by the wit of man.

W. B. Smith, quoted in *Single Variable Calculus*, by James Stewart.

1. (9 points) Which of the following show  $y$  as a function of  $x$ ? Also indicate which do not.



(a)



(b)

$x$	1	2	3	1	5
$y$	-2	4	6	-8	2

(c)

2. (10 points) Let  $f(x) = 1 - 2x^2$ .

(a) (2 points) Compute  $f(3)$ .

(b) (4 points) Compute  $f(x + 3)$  and simplify your answer.

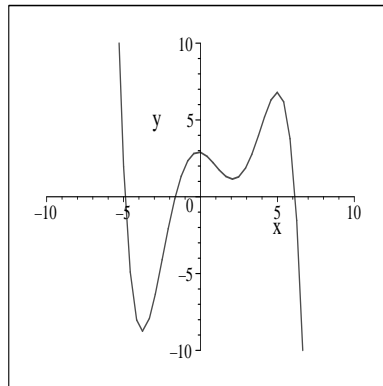
(c) (4 points) Determine the average rate of change of  $f$  over the interval  $[-1, 3]$ .

3. (5 points) Find an equation of the line through  $(-4, 2)$  and  $(3, 1)$ .

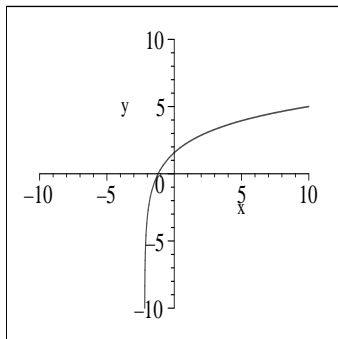
4. (10 points) A credit card has a balance of \$1000. Assume that interest accrues at an annual rate of 14% (typical for credit cards) and no payments are made.
- What will the balance be after 3 years?
  - What is the doubling time for this credit card?

5. (15 points) The function  $f$  is graphed below.

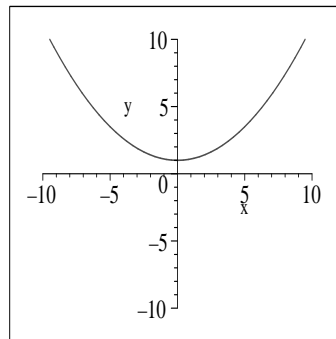
- (3 points) Determine (approximately)  $f(2)$  from the graph.
- (6 points) On the same set of axes, sketch the graph of  $f(x + 1) + 2$ .
- (3 points) Assume that  $f$  is a polynomial. What is the minimum possible degree of  $f$ ?
- (3 points) Again assuming that  $f$  is a polynomial, determine whether its leading coefficient is positive or negative.



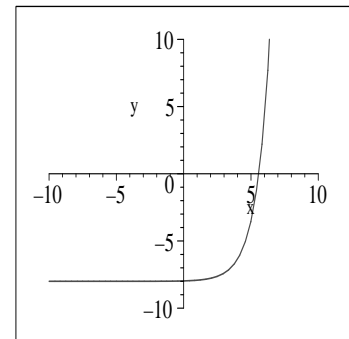
6. (6 points) Three functions are graphed below. Which has the greatest average rate of change over the interval  $[2, 6]$ ? Which has the smallest average rate of change over the interval  $[-2, 6]$ ?



$y = f(x)$



$y = g(x)$



$y = h(x)$

7. (5 points) Let  $f(x) = x^2 + 3x - 1$  and let  $g(x) = \sqrt{x - 1}$ . Compute  $f(g(x))$ .

8. (10 points) Solve for  $y$  algebraically.

(a)  $125e^{0.06y} = 75$ .

(b)  $3 \cdot 5^y = 10 \cdot 2^y$ .

9. (5 points) The intensity  $I$  of light is inversely proportional to the square of the distance  $d$  from the light's source.

(a) Express  $I$  as a function of  $d$ .

(b) If the intensity is 60 units when the distance is 10 meters, what is the intensity when the distance is 25 meters?

10. (15 points) Determine each limit and explain how you arrived at your answer.

(a)  $\lim_{x \rightarrow \infty} \frac{1000x^2}{e^{0.01x}}$

(b)  $\lim_{x \rightarrow \infty} \frac{3e^x + x^3}{2e^x - 7x + 5}$

(c)  $\lim_{x \rightarrow -\infty} 5x^7 - 6x^4 + 2x^3 + 81x - 135$

11. (10 points) True or False.

- (a) \_\_\_\_\_ If interest accrues at an annual rate of 6%, then after 2 years, exactly 12% will have accrued.
- (b) \_\_\_\_\_ A line with positive slope rises as  $x$  increases.
- (c) \_\_\_\_\_ The graph of  $y = -f(x)$  is the graph of  $y = f(x)$  reflected across the  $y$ -axis.
- (d) \_\_\_\_\_ If  $x$  and  $y$  are positive real numbers, then  $\ln(x + y) = \ln(x) + \ln(y)$ .
- (e) \_\_\_\_\_ If  $x$  and  $y$  are positive real numbers, then  $\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$ .
- (f) \_\_\_\_\_ If  $y$  is proportional to  $x$ , then  $x$  is also proportional to  $y$ .
- (g) \_\_\_\_\_ The graph of  $y = f(x) + 1$  is the graph of  $y = f(x)$  shifted right one unit.
- (h) \_\_\_\_\_ The graph of  $y = f(x + 1)$  is the graph of  $y = f(x)$  shifted up one unit.
- (i) \_\_\_\_\_ The graph of  $y = f(x + 1)$  is the graph of  $y = f(x)$  shifted right one unit.
- (j) \_\_\_\_\_ The graph of  $y = f(x) + 1$  is the graph of  $y = f(x)$  shifted up one unit.

12. (5 points) Bonus! Show algebraically that  $\ln(x^y) = y \ln(x)$  for real numbers  $x$  and  $y$ ,  $x > 0$ .