MTH 139-02

Final Exam **Tuesday, December 16, 2003**

Name:_____

Remember to **show your work**. Unsupported solutions will receive no credit.

Computation (40 points)

1. (10 points) Differentiate. You do not need to simplify your answers.

   (a) $f(x) = 4x^5 - 3x^7 + 2 + x^{-5/3}$

   (b) $F(x) = \ln(x^2 + 1)$

   (c) $f(x) = \dfrac{5x + 1}{x^2 - 2}$

   (d) $f(x) = \dfrac{1}{x}$

   (e) $g(x) = x^5(4x^3 - 7x^2 + 9)^4$

2. (2 points) Compute $\displaystyle\sum_{i=3}^{6} i^2$.

3. (10 points) Integrate. In the case of definite integrals, you must show all steps; a numerical answer from your calculator is **not** sufficient. (That is, you must convince me you know how to find the definite integrals without your calculator.)

   (a) $\int (x^2 - 4x^3)dx$

   (b) $\int_1^{25} \frac{1}{\sqrt{x^5}}dx$

   (c) $\int x^3(x^4 + 4)^8 dx$

   (d) $\int e^{-7x}dx$

   (e) $\int_0^5 \frac{2x}{x^2 + 1}dx$

NOTE: For the rest of the exam, you may use your calculator to compute any definite integrals that arise.

4. (2 points) Solve $24(0.8)^{t/6} = 12$ for t.

5. (2 points) Find the average rate of change of $F(x) = \ln(x^2 + 1)$ on the interval $[0, 5]$.

6. (8 points) Compute each sum, if possible. If it is not possible, explain why not.

   (a) $3 + 3 \cdot 1.5 + 3 \cdot (1.5)^2 + 3 \cdot (1.5)^3 + \ldots + 3 \cdot (1.5)^{17}$.

   (b) $2 + \dfrac{2}{5} + \dfrac{2}{25} + \dfrac{2}{125} + \ldots$

   (c) $1 - \dfrac{1}{4} + \dfrac{1}{16} - \dfrac{1}{64} + \ldots$

   (d) $1 + 1.2 + (1.2)^2 + (1.2)^4 + \ldots$

7. (4 points) Let $f(x) = 2x^3 + 3x^2 - 72x + 20$. Use techniques of calculus to find the local and absolute extrema of f on $[-4, 4]$. (I.e., don't just use your calculator!)

8. (2 points) Find the average value of $f(x) = \dfrac{2x}{x^2 + 1}$ on $[0, 5]$.

Concepts (40 points)

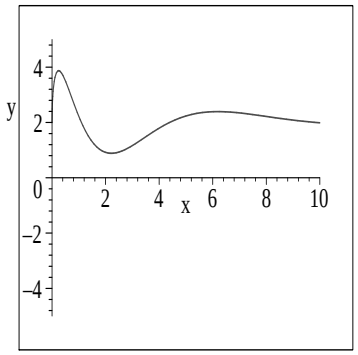
9. (7 points) The graph of $y = f(x)$ is shown below.

   (a) (2 points) Put a small square around the local maxima of f, and put a small circle around the local minima of f.

   (b) (2 points) Draw a small triangle around the inflection points of f.

   (c) (1 point) On the graph of f, draw a small X through the points at which f' is a maximum or a minimum.

   (d) (2 points) Sketch a graph of $y = f'(x)$ on the same set of axes.



$y = f(x)$

10. (3 points) Estimate the rate of change of $f(x) = 5x^2$ at $x = 2$ using $h = 0.01$. Compare this to the actual rate of change of f at $x = 2$.

11. (3 points) Find an equation of the tangent line to the graph of $y = f(x)$, where $f(x) = \ln(x)$, at $x = 1$. Sketch a graph illustrating both f and this tangent line.

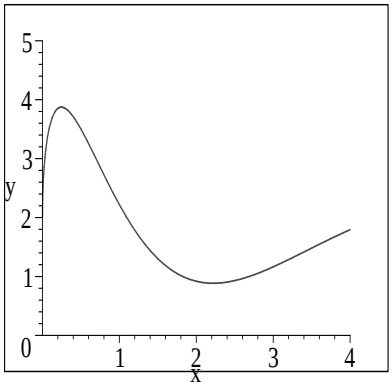
12. (3 points) If $F(0) = 2$ and $F'(x) = \dfrac{x^3}{4}$, what is $F(4)$?

13. (3 points) What is the relationship among Numbers 1b, 3e, 5, and 8?

14. (3 points) Suppose that the average value of f on the interval $[1, 4]$ is 7. What is $\int_1^4 f(x)dx$? How do you know?

15. (3 points) Suppose that the average rate of change of F on the interval $[1, 4]$ is 7. What is the net change in F over the interval $[1, 4]$? How do you know?

16. (2 points) On the graph of $y = f(x)$ shown below, illustrate the average value of f over $[0, 3]$.



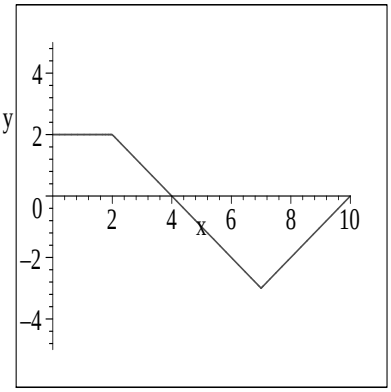
17. (3 points) Use the **definition** of the derivative to find the derivative of $f(x) = -3x^2$.

18. (6 points) Define $F(x)$ by $F(x) = \int_0^x f(t)dt$, where $y = f(x)$ is the graph shown.

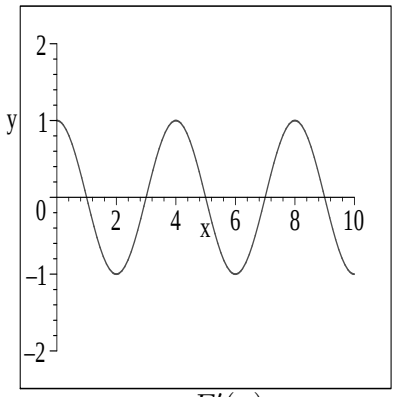
(a) Illustrate $F(5)$ graphically.

(b) What is the numerical value of $F(5)$?

(c) What is $F'(5)$?



19. (2 points) The graph shown below is the graph of $y = F'(x)$. Given that $F(0) = 0$, for what other values of b is $F(b) = 0$?



$y = F'(x)$

20. (2 points) Suppose that the graph of f is concave down at $x = c$. What can you conclude about f' at $x = c$?

Applications (20 points)

21. (5 points) You take out a $10,000 business loan to start your new doorknob polishing company. If the interest rate is 12% annually, compounded monthly, and you make monthly payments of $150, how long will it take to pay off the loan? You must show your work to receive credit!

22. (5 points) The amount of a certain drug in a patients's bloodstream t hours after taking the drug is given by $A(t) = te^{-t/5}$. At what time is the amount in the bloodstream the greatest?

23. (5 points) A police officer's radar gun clocks a motorist travelling

$$v(t) = \frac{1}{3600}\left(\frac{10(t^2+2)}{t^2+1} + 60\right)$$

miles per second t seconds after the officer begins monitoring. How far does the motorist travel in one minute?

24. (5 points) Suppose that the federal government is giving an \$8 billion tax rebate. Assuming that people spend 75% of the money they receive (from any source), how much additional spending is generated?

Congratulations! I hope to have grades posted by Wednesday afternoon. Have a great break and a wonderful Christmas!