

# In-Class Assignment 1

MATH 139-02  
Friday, January 30, 2004

**Directions:** Work neatly on a separate sheet of paper. Your group will hand in one write-up with everyone's name on it. **DO NOT** fold the corner over to hold everything together!

Work together on each problem; do not delegate different problems to different people.

1. Sketch a graph of a function with the specified properties, if possible.
  - (a) The graph of the function switches from being increasing and concave up to increasing and concave down at a point  $P$ .
  - (b) The graph of the function switches from being increasing and concave up to decreasing and concave up at a point  $P$ .
  - (c) The graph of the function switches from being increasing and concave down to decreasing and concave down at a point  $P$ .
  - (d) The graph of the function switches from being increasing and concave up to decreasing and concave down at a point  $P$ .
2. We have seen that if interest accrues annually, we can compute our balance after  $t$  years using the formula  $B(t) = P(1+r)^t$ , where  $P$  is our principal (original deposit) and  $r$  is our interest rate expressed as a decimal. We arrived at this by observing that no matter how much we begin a year with, at the end of the year we have  $1+r$  times as much. (This observation will come in handy below.)

In this exercise you will investigate what happens when interest is compounded more frequently. Note that the interest rate is prorated for interest accumulated before the year is up. For example, if the annual rate is 12%, the amount compounded after 1 month is only 1%.

- (a) Suppose interest accrues quarterly (four times per year).
  - i. What is the balance at the end of the first quarter (in terms of  $P$  and  $r$ )? [Hint: the interest rate for 1 quarter is  $r/4$ .]
  - ii. What is the balance at the end of the second quarter?
  - iii. What is the balance at the end of the third quarter?
  - iv. What is the balance at the end of the first year? (Four quarters)
  - v. What is the balance after  $t$  years?
- (b) Now suppose interest accrues monthly (12 times per year).
  - i. What is the balance at the end of the first month?
  - ii. What is the balance at the end of the fifth month?
  - iii. What is the balance at the end of the first year?
  - iv. What is the balance after  $t$  years?
- (c) Suppose that interest accrues  $n$  times per year.
  - i. What is the balance at the end of the first compounding period?
  - ii. What is the balance at the end of the second compounding period?
  - iii. What is the balance at the end of the first year?
  - iv. What is the balance after  $t$  years?
- (d) Begin with a balance of \$1 and an interest rate of 100% (so that your balance will double in one year).
  - i. What is your balance after one year if interest is compounded quarterly?
  - ii. Monthly?
  - iii. Daily?
  - iv. Hourly?
  - v. Every minute?
  - vi. What trend do you observe?