In-Class Assignment 2

MATH 141

Directions: Work neatly on a separate sheet of paper. Your group will hand in one write-up with everyone's name on it. **DO NOT** fold the corner over to hold everything together! Your final write-up should be very neat and well-written. Remember to use complete sentences as appropriate.

Work together on each problem; do not delegate different problems to different people.

1. Suppose that a car travels along a straight road with its position given by $x(t) = 6t - t^2$ miles from home t hours after it sets out.
   (a) Find the average velocity between hours 2 and 3.
   (b) Find the average velocity from 2 to 2.5 hours.
   (c) Find the average velocity from 2 to 2.1 hours.
   (d) Find the average velocity from 2 to 2.01 hours.
   (e) Find the average velocity from 1.9 to 2 hours.
   (f) Find the average velocity from 1.99 to 2 hours.
   (g) Find the average velocity from 2 to $2 + h$ hours (here, h is an unknown time, possibly negative, added to the 2 hours).
   (h) Based on your work above, what is the velocity when $t = 2$ hours?
   (i) Find an equation of the line L that (1) has slope equal to the velocity you found above and (2) passes through the point $(2, x(2))$.
   (j) Sketch the graphs of $x(t) = 6t - 6t^2$ and the line L on the same set of axes. This is the **tangent line to the graph of x at $t = 2$.**
   (k) Write a brief explanation of the connection between the slope of the tangent line and the velocity.
2. Consider the function $x(t) = \sqrt{t}$.
   (a) Sketch the graph of $x(t)$ as accurately as you can.
   (b) Locate the point $(1, 1)$ on the graph of $x(t) = \sqrt{t}$.
   (c) Sketch the line through the points on the graph with t-coordinates 0 and 1. What is its slope? (This line is called a **secant** line.)
   (d) Sketch the secant line through the points on the graph with t-coordinates 0.5 and 1. What is its slope?
   (e) Sketch the secant line through the points on the graph with t-coordinates 0.9 and 1. What is its slope?
   (f) Sketch the secant line through the points on the graph with t-coordinates 0.99 and 1. What is its slope?
   (g) What is your best estimate for the slope of the line tangent to the graph and passing through $(1, 1)$?
   (h) Using your estimate from above, find an equation of the tangent line to $x(t) = \sqrt{t}$ at $(1, 1)$.
   (i) Use the tangent line to estimate $\sqrt{1.03}$. Compare this to your calculator's value for $\sqrt{1.03}$.
   (j) Use the tangent line to estimate $\sqrt{6}$. Compare this to your calculator's value for $\sqrt{6}$. Explain your results graphically.