In-Class Assignment 9: Arc Length

MATH 142

Directions: Work neatly on a separate sheet of paper. Your **group** will hand in one write-up with everyone's name on it. **DO NOT** fold the corner over to hold everything together!

Work together on each problem; do not delegate different problems to different people.

1. Sketch a graph of $f(x) = \frac{4}{3}x^{3/2}$. Make sure that what you graph includes the interval $[0, 2]$. Sketch the graph fairly large so that you can add to it without having it get ridiculously cluttered.
2. Subdivide $[0, 2]$ into $n = 2$ subintervals of equal width. Draw a line segment between the endpoints of the graph on each subinterval and compute their lengths. What is their combined length?
3. Repeat part 2 with $n = 4$.
4. Now imagine a partition of $[0, 2]$ into n subintervals of equal width using our usual partition. What is the distance between the endpoints of the graph over the subinterval $[x_{i-1}, x_i]$?
5. What is the total distance you get when you add the lengths of all n line segments?
6. Here is where we need to recall the Mean Value Theorem, restated in terms of what we need here: if f is continuous on $[x_{i-1}, x_i]$ and differentiable on (x_{i-1}, x_i), then there is a number $c_i \in [x_{i-1}, x_i]$ such that $f'(c_i)(x_i - x_{i-1}) = f(x_i) - f(x_{i-1})$, or $f'(c_i)\Delta x = f(x_i) - f(x_{i-1})$. Make this substitution for $f(x_i) - f(x_{i-1})$ in your expression for the total length and simplify the result.
7. You know what's coming: the limit as $n \to \infty$ of your result from number 6 is called the **arc length** of f from 0 to 2. Express that limit as an integral.
8. Use your integral from above to find the arc length of the graph of $f(x) = \frac{4}{3}x^{3/2}$ on $[0, 2]$.
9. Now generalize: use the ideas from above to develop a general formula for the arc length of $y = f(x)$ over an interval $[a, b]$.