In-Class Assignment 12: Taylor Series

MATH 142

Directions: Work neatly on a separate sheet of paper. Your **group** will hand in one write-up with everyone's name on it. **DO NOT** fold the corner over to hold everything together!

Work together on each problem; do not delegate different problems to different people.

1. Find the Taylor series for each function below. Take advantage of known series whenever possible.
   (a) $f(x) = \tan x$ centered at $a = 0$. (Just the first four terms.)
   (b) $f(x) = e^{2x}$
   (c) $f(x) = \sin x + \cos x$
   (d) $f(x) = e^x \sin x$
   (e) $f(x) = x^4 + 3x^3 - 7x + 1$
   (f) $f(x) = \arcsin x$
   (g) $f(x) = x\sqrt{1+x}$
2. Evaluate your series for $\arcsin x$ (1(f) above) at $x = 1$ to discover an identity involving π.
3. Recall that the **imaginary unit** i has the property that $i^2 = -1$.
   (a) Using the series for e^x, write out the series for $e^{i\theta}$.
   (b) Separate the real and imaginary parts: put the parts with no i together and the parts that do have an i together. What familiar series (plural) do you see?
   (c) Use your work above to evaluate $e^{i\pi}$.
4. Using series, approximate $\int_0^2 e^{-x^2}\,dx$. (It is actually **necessary** to use series for this since there is no elementary antiderivative for e^{-x^2}.)