

MATH 150

Exam 3

Friday, November 6, 2020

Name: _____

Remember to **show your work**. Unsupported solutions will receive no credit.

The calculus is the greatest aid we have to the appreciation of physical truth in the broadest sense of the word.

– W. F. Osgood, quoted in *Bulletin American Mathematical Society*

- (10 points) Given that $\sin(\theta) = \frac{3}{7}$, determine each quantity below **exactly** (i.e., don't use your calculator to approximate a decimal answer).
 - $\cos(\theta) =$ _____
 - $\sin(2\theta) =$ _____
 - $\tan(\theta) =$ _____
- (30 points) Differentiate each function. **You do not need to simplify your answers.**
 - $f(x) = e^{\cos(x)}$
 - $f(x) = x^2 \sin(x)$
 - $f(x) = \arctan(e^{6x})$
 - $f(x) = \tan(4x^2)$
 - $f(x) = (\ln(x+2) + \sec(5x^2))^4$
 - $f(x) = x^x$
- (10 points) Saruman is able to grow his Uruk-hai army at a rate of 18% every 14 days. How long does it take for Saruman to double the size of his army?
- (10 points) Grond is swung back and forth in an attempt to breach the gates of Minas Tirith. At time t seconds, Grond's snout is $x(t) = 12 + 12 \cos\left(\frac{\pi t}{6}\right)$ meters from the gates of Minas Tirith. Find the speed of Grond's snout at time $t = 1$ s.
- (20 points) True or False/fill-in.
 - _____ The domain of $\cos(x)$ is $(-\infty, \infty)$.
 - _____ The domain of $\tan(x)$ is $(-\infty, \infty)$.
 - _____ $\sqrt{x^2 + 4} = x + 2$ for all $x \geq 0$.
 - _____ $\ln(ab) = \ln(a) + \ln(b)$ for all $a, b > 0$.
 - _____ $\ln(a+b) = \ln(a) + \ln(b)$ for all $a, b > 0$.
 - A sample of an element with a half-life of 12s has _____% remaining after 24s.
 - $\lim_{x \rightarrow \infty} \frac{e^{5x}}{3 + 2e^{5x}} =$ _____
 - $\lim_{x \rightarrow \infty} \frac{x^{2020}}{1.00001^x} =$ _____
 - $\lim_{x \rightarrow \infty} \frac{4x^6 - 3x^4 + 2x^2 - 8x + 9}{7x^6 - 2x^5 + 3x^3 + 2x - 4} =$ _____
 - $\lim_{x \rightarrow \infty} e^{-3x} =$ _____
- (10 points) A surveyor measures an angle of 42° between the ground and the top of a building. If she is 50 feet from the building, how tall is the building?
- (10 points) Find an equation of the tangent line to $f(x) = \sin(4x)$ at $x = \frac{\pi}{6}$.
- BONUS!!!** (10 points) Express $\cos(3\theta)$ in terms of $\cos(\theta)$.