MATH 150

Final Exam **Tuesday, December 8, 2020**

Name:______________________

Remember to **show your work**. Unsupported solutions will receive no credit.

> **Nothing takes place in the world whose meaning is not that of some maximum or minimum.**
> **– Leonhard Euler**

Formulas

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

	Volume	Surface Area
Cylinder	$\pi r^2 h$	$2\pi rh + 2\pi r^2$
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Cone	$\frac{1}{3}\pi r^2 h$	$\pi r\ell + \pi r^2$

Computation (60 points)

In this section, you may not compute integrals on your calculator (except to check). You must integrate by hand to receive **any credit at all**.

1. (15 points) Differentiate. You do not need to simplify your answers.

   (a) $f(x) = 5x^6 - 4x^3 + 7x + 2$

   (b) $f(x) = e^{3x} \sin x$

   (c) $f(x) = \dfrac{\sin x}{3x^2 - 4}$

2. (5 points) Find an equation of the tangent line to the graph of $f(x) = 2x + \tan(3x) - 1$ at $x = 0$.

3. (5 points) Compute the sum $\displaystyle\sum_{k=2}^{6} \frac{3k}{k+4}$. You do not need to simplify.

4. (20 points) Integrate. In the case of definite integrals, you must show all steps; a numerical answer from your calculator is **not** sufficient. (That is, you must convince me you know how to find the definite integrals without your calculator.)

   (a) $\displaystyle\int_{-1}^{2} 6x^3 dx$

   (b) $\displaystyle\int \frac{6}{x} + e^{4x} dx$

(c) $\int_0^1 (x+1)(3x^2+6x+4)^{12} dx$

(d) $\int \sin(x) - \frac{4}{x^2} dx$

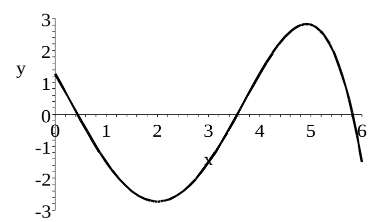
5. (5 points) Compute: $\lim_{x \to \infty} \frac{e^{3x}}{e^{3x}+x}$

6. (10 points) Compute $\int_1^6 x^2 dx$ using the **definition** of the definite integral.

Concepts (60 points)

7. (5 points) If $f'(x) = \frac{3x^2}{\ln(x)+6}$ and $g(x) = f(x) - 4$, what is $g'(2)$?

8. (10 points) The graph of $y = f(x)$ is shown below. Sketch a graph of $y = f'(x)$ on the same set of axes.



$y = f'(x)$

9. (10 points) Let $f(x) = x^4 - 8x^2 + 3$. Use techniques of calculus to find the absolute extrema of f on $[0, 3]$. (I.e., don't just use your calculator!)

10. (5 points) Find the area of the region below the graph of $f(x) = 16 - x^2$ between $x = 0$ and $x = 4$.

11. (10 points) For $f(x) = \sqrt{x}$, use the tangent line to estimate $\sqrt{9.01}$.

12. (10 points) Sand drains through an hourglass at the rate of 2cm^3 per minute and forms a conical pile in which the height and radius are always equal. At what rate is the radius of the pile changing when the volume is 9π cm^3?

13. (10 points) True or False. Assume f is differentiable everywhere.

(a) __________ If $f'(a) = 0$ and $f''(a) > 0$, then $f(a)$ is a local maximum.

(b) __________ If $F(x)$ is an antiderivative for $f(x)$, then $F'(x) = f(x)$.

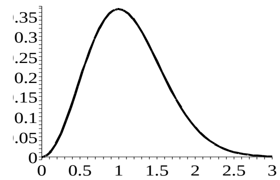
(c) __________ If $f(2) = 10$ and $f(5) = 4$, then $f'(c) = -2$ for some $c \in [2, 5]$.

(d) __________ If $f'(x) > 0$ for all x, then $f(4)$ is less than $f(7)$.

(e) __________ A function f is concave up if the slope of f is decreasing.

Applications (30 points)

14. (10 points) The graph below shows the velocity of a proton in m/s after t seconds have elapsed. Estimate how far the proton travels for $t \in [1, 3]$ using 4 subintervals.



15. (10 points) In a certain chemical reaction, the concentration C of a chemical is given by $C = \dfrac{4t}{2t+1}$. The **rate of reaction** is the derivative of C with respect to time, $\dfrac{dC}{dt}$. Find the rate of reaction at $t = 3$ seconds.

16. (10 points) Bilbo is designing barrels for the comfort of anyone who has to ride inside of them. He wants the barrel to be a cylinder, and for storage reasons, its circumference $C = 2\pi r$ plus its length ℓ must be no more than 4m. What is the largest possible volume of a such a barrel?

Bonus! (10 points)

17. (10 points) The graph of $f'(x)$ is given to the right. Draw a possible graph of $f(x)$ on the same set of axes.