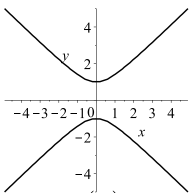
MATH 150

Exam 1 **Wednesday, September 16, 2020**

Name:________________________

Remember to **show your work**. Unsupported solutions will receive no credit.

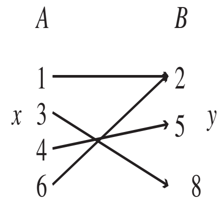
1. (10 points) **(a)** Which of the following show y as a function of x? Also indicate which do not. **(b)** For each one that is a function, also indicate whether it has an inverse.



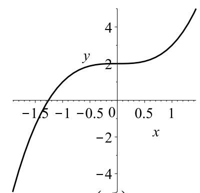
(a)

x	1	2	3	4	5
y	-2	4	-2	-8	2

(b)



(c)



(d)

Solution: (a) fails the vertical line test and is not a function. (b) has no x-values used twice, so it represents a function. (c) has no x-values used twice, so it represents a function. (d) passes the vertical line test, so it represents a function.

2. (5 points) Let $f(x) = 2 + x^2$. Determine the average rate of change of f over the interval $[1, 3]$.

Solution: The AROC is $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{(2 + 3^2) - (2 + 1^2)}{2} = 4$.

3. (5 points) Find an equation of the line through $(2, 1)$ and $(4, -3)$.

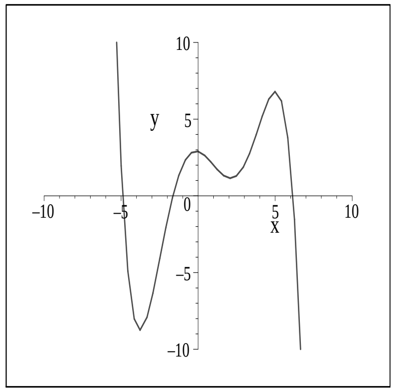
Solution: The slope is $\dfrac{-3 - 1}{4 - 2} = -2$, so we get $y - 1 = -2(x - 2)$. We could also use the other point: $y - (-3) = -2(x - 4)$.

4. (10 points) Use the table below to approximate $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	1.3784	1.4107	1.4139	1.4142	1.4146	1.4177	1.4491

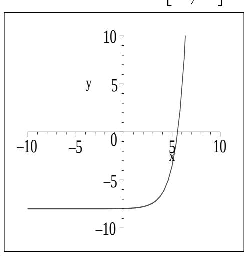
Solution: I will just use the points closest to $x = 2$: $\lim\limits_{x \to 2} \dfrac{f(2.001) - f(2)}{2.001 - 2} = \dfrac{1.4146 - 1.4142}{0.001} = 0.4$.

5. (15 points) The function f is graphed below. On the same set of axes, sketch the graph of $f(x + 2) - 3$.

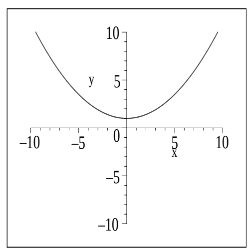


Solution: That's tricky for me on the computer, so I will just say that it should be shifted left 2 and down 3.

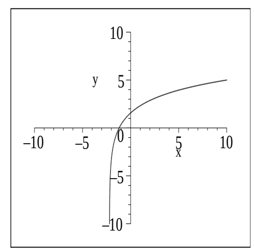
6. (10 points) Three functions are graphed below. Which has the greatest average rate of change over the interval $[2, 7]$? Which has the smallest average rate of change over the interval $[2, 7]$?



$y = f(x)$



$y = g(x)$



$y = h(x)$

Solution: If we draw a line on each graph that connects the points at $x = 2$ and $x = 7$, we see that the first graph has the steepest secant line and the last graph has the shallowest, so the first has the greatest AROC and the last has the smallest.

7. (10 points) What is the effect on the graph of $y = f(x)$ of each transformation?

   (a) Reflect across x-axis $y = -f(x)$

   (b) Reflect across y-axis $y = f(-x)$

   (c) Shift left 1 unit $y = f(x + 1)$

   (d) Shift up 1 unit $y = f(x) + 1$

   (e) Vertical stretch by a factor of 2 $y = 2f(x)$

8. (10 points) Identify each discontinuity of each function and classify it (point, jump, or infinite).

   (a) $f(x) = \dfrac{x^2 + 1}{x - 3}$ This has an infinite discontinuity at $x = 3$.

   (b) $f(x) = \dfrac{(2x - 3)(4x + 1)}{(4x + 1)(x - 2)}$ Removable discontinuity at $x = -1/4$; infinite discontinuity at $x = 2$.

9. (10 points) Compute each limit.

(a) $\lim_{x\to 5} \dfrac{x^2 - 2x + 3}{x^2 + 1}$

Solution: $\lim_{x\to 5} \dfrac{x^2 - 2x + 3}{x^2 + 1} = \dfrac{5^2 - 2(5) + 3}{5^2 + 1}$

(b) $\lim_{h\to 0} \dfrac{(3 + h)^2 - 3^2}{h}$

Solution: $\lim_{h\to 0} \dfrac{(3 + h)^2 - 3^2}{h} = \lim_{h\to 0} \dfrac{9 + 6h + h^2 - 9}{h} = \lim_{h\to 0} \dfrac{h(6 + h)}{h} = \lim_{h\to 0} 6 + h = 6$.

Solution:

(c) $\lim_{x\to 3} \dfrac{x^2 - 7x + 12}{x - 3}$

Solution: $\lim_{x\to 3} \dfrac{x^2 - 7x + 12}{x - 3} = \lim_{x\to 3} \dfrac{(x - 3)(x - 4)}{x - 3} = \lim_{x\to 3} x - 4 = -1$.

(d) $\lim_{x\to\infty} \dfrac{3x^2 - 6x + 1}{2x^2 + 5x + 2}$

Solution: $\lim_{x\to\infty} \dfrac{3x^2 - 6x + 1}{2x^2 + 5x + 2} = \dfrac{3}{2}$ (ROTLC)

10. (5 points) The **Lorentz factor** is a function that shows up in relativity; $f(v) = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, where c is the speed of light. Since nothing can go faster than light, any limit for v approaching c must come from the left. Find $\lim_{v\to c^-} \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

Solution: $\lim_{v\to c^-} \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \infty$ since the denominator goes to 0^+ and the numerator goes to 1.

11. (10 points) Solve the inequality $\dfrac{(x - 3)^2(x + 2)}{2x + 5} \geq 0$ and express your answer in interval notation.

Solution: There are only three places the expression on the left can change sign: $-5/2, -2$, and 3. Putting those on a number line and checking for the signs in each interval gives us that the expression is greater than or equal to 0 on $(-\infty, -5/2) \cup [-2, \infty)$ (and negative on $(-5/2, -2)$). Note that $-5/2$ is not in the domain, so it can't be part of the solution, while -2 is and gives 0. Also, while 3 appears on our number line, because $x - 3$ is squared it can never give us a negative. We do get 0 at $x = 3$, so we include 3 and don't need to break up our intervals there.