MATH 150

Exam 2 **Monday, October 12, 2020**

Name:_____

Remember to **show your work**. Unsupported solutions will receive no credit.

> **The calculus is the greatest aid we have to the appreciation of physical truth in the broadest sense of the word.**
> **– W. F. Osgood, quoted in Bulletin American Mathematical Society**

1. (10 points) Use the **definition** of the derivative to show that $\frac{d}{dx}(4x^2) = 8x$. **You must use the definition and show your work to receive credit!**

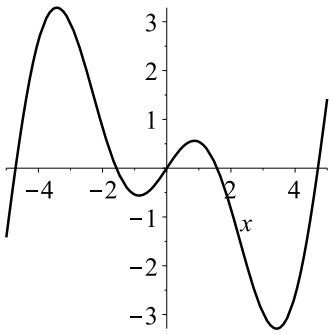
2. (15 points) Calculate the derivative of each function. **DO NOT** just write down an answer; I will need to see your steps in order to award credit.

   (a) $f(x) = \sqrt[3]{x}$.

   (b) $f(x) = x^2(6x^4 - 4x^2 + 1)^9$.

   (c) $f(x) = \dfrac{3x}{4x^3 - 8}$

3. (10 points) Below is the graph of $y = f(x)$. Sketch the graph of $y = f'(x)$ on the same set of axes. **NOTE:** a crude sketch is fine; I just want the general shape of the graph of f'.



4. (20 points) Let $f(x) = x^3 - 6x^2$.

   (a) Find the critical points and local extrema of f.

   (b) Find the inflection points of f.

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5. (20 points) Suppose that f is a differentiable function with values as in the table.

x	$f(x)$	$f'(x)$
-2	4	-2
0	3	0
1	4	6
3	7	0
4	0	-3

   (a) What are the critical points of f according to the table?

   (b) Identify each critical point as a local maximum, a local minimum, or neither.

   (c) Find an equation of the tangent line to the graph of f at $x = 1$.

   (d) Approximate $f(1.1)$.

6. (15 points) True or False/fill-in. Assume g is a function that is continuous and differentiable everywhere.

   (a) _____ If $f(x)$ is a polynomial of degree 5, then f can have as many as 6 extrema.

   (b) _____ The slope of the tangent line to the graph of $y = f(x)$ at $x = a$ is $f'(a)$.

   (c) _____ The function $f(x) = x^2 - 4x + 3$ has a horizontal tangent line in $[1, 3]$.

   (d) _____ If the points $(1, 3)$ and $(4, 6)$ are on the graph of g, then $g'(x)$ **must** equal 2 somewhere in the interval $(1, 3)$.

   (e) _____ If the points $(1, 3)$ and $(4, 6)$ are on the graph of g, then $g'(x)$ **must** equal 1 somewhere in the interval $(1, 3)$.

7. (10 points) Find an equation of the tangent line to $f(x) = x + x^{1/3}$ at $x = 1$.

8. (10 points) **BONUS!!!** Use the definition of the derivative to prove that

$$((f + g)(x))' = f'(x) + g'(x).$$