Homefun 15

MATH 150
15 points

Directions: Work in groups of 2 to 4 in class and then finish outside of class as necessary. Each group should submit **ONE** solution page for the group. (Be sure everyone's name is on it!)

NOTE: There is further supporting information in 5.4 of your text.

Contrast:
Linear functions: A change in x by 1 results in a change in y **adding** a constant m: $y = mx + b$.
Exponential functions: A change in x by 1 results in a change in y by **multiplying by** a constant b: $y = Ab^x$.
Linear functions change by a fixed amount, while exponential functions change by a fixed **percentage**.

1. **Compound growth formula:** If $Q(t)$ grows at r percent (expressed as a decimal) compounded n times per time unit t, then $Q(t) = Q_0\left(1 + \frac{r}{n}\right)^{nt}$, where Q_0 is the initial quantity.

   (a) Interest compounds exponentially. If \$2500 is deposited initially and the interest rate is $r = 5\%$, what is the balance after 15 years if it is compounded monthly? [Ans: \$5284.26]

   (b) Shockingly, $\lim_{n\to\infty} Q_0\left(1 + \frac{r}{n}\right)^{nt} = Q_0e^{rt}$. That is, continuously compounded growth (or decay) at a rate of r percent (expressed as a decimal) results in $Q(t) = Q_0e^{rt}$. Using the 5% and \$2500 from part (a), what is the balance after 15 years if growth is continuous instead of monthly? **NOTE:** The r in such an expression is called the **continuous growth rate**. [Ans: \$5292.50]

2. Useful fact (aka a Theorem): the doubling time (or half-life) of an exponential function is constant.

   (a) Verify this for $A(t) = 12e^{0.4t}$ by finding how long it takes to get to 24 and then how much longer to get to 48. What about 96? [Ans: 1.733]

   (b) Verify the half life formula for $A(t) = 20e^{-0.08t}$ by finding how long it takes to get to 10 and then how much longer to get to 5. What about 2.5? [Ans: 8.664]

3. Determine the half-life of a radioactive material that loses 10% of its mass every 2 hours. [Hint: if 10% is lost every two hours, what percentage remains after every two-hour period? Use that as the base of your exponential function.] [Ans: 13.158 hours.]