MATH 152

Today

1. Questions/WeBWorK
2. 5.2 The Definite Integral

Goals:

1. 5.2 The Definite Integral (Understand the definition of the definite integral and its relationship to the area/distance problem)

Where is today's material used?

1. Physics: distance traveled by a particle (among many others)
2. Chemistry: fraction of gas molecules that can participate in a reaction (among many others)
3. Economics: finding total cost given marginal cost (among many others)
4. Any discipline that includes a notion of accumulated change.

5.2: The Definite Integral

1. $A = bh$ and $d = rt$ have the same form, which connects (rectangle) areas to net change.
2. We can approximate curving functions with flat functions.
3. Our standard notations:
   (a) Interval $[a, b]$ with $a < b$ is subdivided into n subintervals of equal width $\Delta x = \dfrac{b-a}{n}$.
   (b) Subdivision points are $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$.
   (c) $m_i, M_i \in [x_{i-1}, x_i]$ such that $f(m_i)$ is a global min and $f(M_i)$ is a global max on $[x_{i-1}, x_i]$.

(d) x_i^* is a point of our choosing in $[x_{i-1}, x_i]$.

(e) $\sum_{i=m}^{n} f(i) = f(m) + f(m+1) + \ldots + f(n-1) + f(n)$.

4. A **Riemann sum** is a sum of the form

$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i = f(x_1^*)\Delta x_1 + f(x_2^*)\Delta x_2 + \ldots + f(x_n^*)\Delta x_n.$$

5. The **definite integral of f from a to b** is $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$, provided the limit exists.

6. **Theorem:** If f is continuous on $[a, b]$ (or has only finitely many jump or point discontinuities), then $\int_a^b f(x)dx$ is defined (the limit exists).

7. **Note**: $\int_a^b f(x)dx = \int_a^b f(y)dy = \int_a^b f(t)dt = \cdots$

8. Examples: 5.2, p. 279: 13, 15, 19

Next Time

1. 5.3 The Evaluation Theorem and the Net Change Theorem

2. Turn in WeBWorK 5.2, Set04-DefiniteInt: 3, 5