

# MATH 152

## Today

1. Questions/WeBWorK
2. 5.5  $u$ -Substitution

### Goals:

1. 5.5  $u$ -Substitution (Understand how to use  $u$ -substitution and that  $u$ -substitution is used to reverse the Chain Rule)

## Where is today's material used?

1. Physics: distance traveled by a particle (among many others)
2. Chemistry: fraction of gas molecules that can participate in a reaction (among many others)
3. Economics: finding total cost given marginal cost (among many others)
4. Any discipline that includes a notion of accumulated change.

## 5.5: $u$ -Substitution

1. **Theorem (The Chain Rule):** If  $f$  and  $g$  are differentiable functions and  $F = f \circ g$  is their composition, then  $F$  is differentiable and

$$F'(x) = f'(g(x))g'(x).$$

In Leibniz notation: if  $y = f(u)$  and  $u = g(x)$ , then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}.$$

2. **Theorem ( $u$ -substitution):** If  $u = g(x)$  be a differentiable function whose range is an interval  $I$ , and assume that  $f$  is continuous on  $I$ . Then  $\int f(g(x))g'(x)dx = \int f(u)du$ .

3. **Theorem:** If  $u = g(x)$  has a continuous derivative on  $[a, b]$  and  $f$  is continuous on  $f(g([a, b]))$  then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du.$$

4. Examples: 5.5, p. 306: 12, 19, 22, 30, 34, 45, 62

## Next Time

1. 6.1 Integration by parts
2. Turn in WeBWorK 5.5, Set07-USubstitution: 5, 6