

MATH 152

Today

1. WeBWorK/Questions
2. 6.6 Improper integrals

Goals:

1. 6.6 Improper Integrals (Understand the different types of improper integrals and how to work with them)

Where is today's material used?

1. Physics: distance traveled by a particle (among many others)
2. Chemistry: fraction of gas molecules that can participate in a reaction (among many others)
3. Economics: finding total cost given marginal cost (among many others)
4. Any discipline that includes a notion of accumulated change.

6.6 Improper Integrals

1. Definition:

(a) If f is continuous on $[a, \infty)$, then
$$\int_a^{\infty} f(x)dx = \lim_{b \rightarrow \infty} \int_a^b f(x)dx.$$

(b) If f is continuous on $(-\infty, b]$, then
$$\int_{-\infty}^b f(x)dx = \lim_{a \rightarrow -\infty} \int_a^b f(x)dx.$$

Such integrals are called **improper**. If the limit exists, the integral is said to **converge**; otherwise, it **diverges**.

(c) If f is continuous on $\mathbb{R} = (-\infty, \infty)$, then
$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^{\infty} f(x)dx,$$
 where $c \in \mathbb{R}$, provided both integrals converge.

2. Definition:

(a) If f is continuous on $[a, b)$ and has an infinite discontinuity at $x = b$, then $\int_a^b f(x)dx = \lim_{c \rightarrow b^-} \int_a^c f(x)dx$.

(b) If f is continuous on $(a, b]$ and has an infinite discontinuity at $x = a$, then $\int_a^b f(x)dx = \lim_{c \rightarrow a^+} \int_c^b f(x)dx$.

Such integrals are called **improper**. If the limit exists, the integral is said to **converge**; otherwise, it **diverges**.

(c) If f is continuous on $[a, b]$ except at some $c \in [a, b]$ at which f has an infinite discontinuity, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, provided both integrals converge.

3. **Theorem (The Comparison Theorem):** Let f, g be continuous on $[a, \infty)$ and suppose $f(x) \geq g(x) \geq 0$ on $[a, \infty)$.

(a) If $\int_a^\infty f(x)dx$ is convergent, so is $\int_a^\infty g(x)dx$.

(b) If $\int_a^\infty g(x)dx$ is divergent, so is $\int_a^\infty f(x)dx$.

4. Examples: 6.6, p. 360: 1,2, 5-32, 41-46

Next Time

- 7.1/7.2 Area between curves/volumes of solids
- Turn in WeBWork Set10-Summary: 4 and Set11-Improper 5, 6