MATH 152

Today

1. WeBWorK/Questions
2. 7.1 Area between curves
3. 7.2 Volumes of solids

Goals:

1. 7.1 Area between curves (Understand how to find the area bounded by two functions of x)
2. 7.2 Volumes of solids (Understand how to use integrals to find volumes via cross sections))

Where is today's material used?

1. Physics: distance traveled by a particle (among many others)
2. Chemistry: fraction of gas molecules that can participate in a reaction (among many others)
3. Economics: finding total cost given marginal cost (among many others)
4. Any discipline that includes a notion of accumulated change.

7.1 Area between curves

1. **Theorem:** Let f and g be continuous functions on $[a, b]$ such that $g(x) \leq f(x)$ for all $x \in [a, b]$. Let R be the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$. Then

$$A(R) = \int_a^b (f(x) - g(x))dx.$$

2. Examples: 7.1, p. 369: 5, 10, 14, 18

7.2 Volumes of solids

1. We have volume axioms analogous to the area axioms: Let S be a solid region in space.
   (a) (Existence) There is a non-negative real number $V(S)$, called the volume of S, associated with S.
   (b) (Congruence) If T is a solid region congruent to S, then $V(T) = V(S)$.
   (c) (Dominance) If $T \subseteq S$ and T is a solid region, then $V(T) \leq V(S)$.
   (d) (Unit) The volume of a unit cube is 1.
   (e) (Additivity) If S and T are nonoverlapping solid regions in space (except possibly on their boundaries), then $V(S \cup T) = V(S) + V(T)$.
2. **Theorem:** Suppose that S is a solid region in space with cross-sectional area given by $A(x)$ in the vertical plane perpendicular to the x-axis for each $x \in [a, b]$. Then the volume of S is
$$V(S) = \int_a^b A(x)dx.$$
3. **Definition:** A **solid of revolution** is a solid obtained by revolving a plane region about a line.
4. Examples: 7.2, p. 378: 3, 5, 12

Next Time

1. Finish 7.2
2. 7.3 Volumes by cylindrical shells
3. Turn in WeBWorK 7.1, Set12-Area: 3, 4