MATH 153

Today

1. WeBWorK/Questions
2. 8.1 Sequences

Goals:

1. 8.1 Sequences (Understand the definition of sequences, how to interpret sequence notation, and how to create a formula for a sequence)

Where is today's material used?

1. Sequences are the basis for series, for which we have already seen several applications.

8.1 Sequences

1. **Definition:** An **infinite sequence** is a function $a : \mathbb{Z}^+ \to \mathbb{R}$.
2. Notation: $a(1) = a_1, a(2) = a_2, \ldots, a(n) = a_n, \ldots$. **The subscript tells you which term you are on, not the value of the term.**
3. Notation: We use the shorthand $\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$ to describe the sequence $a_1, a_2, a_3, \ldots$.
4. **Definition:** A sequence $a_1, a_2, a_3, \ldots$ has the **limit** b, written $\lim_{n\to\infty} a_n = b$, if for every $\varepsilon > 0$ there is an integer N such that whenever $n > N$, $|a_n - b| < \varepsilon$. If this is a case, the sequence **converges** or **is convergent**; if not, it **diverges** or **is divergent**.
5. **Definition:** A sequence $\{a_n\}$ is **increasing** if $a_n \leq a_{n+1}$ for all $n \geq 1$. It is **strictly increasing** if $a_n < a_{n+1}$ for all $n \geq 1$.
6. **Definition:** A sequence $\{a_n\}$ is **decreasing** if $a_n \geq a_{n+1}$ for all $n \geq 1$. It is **strictly decreasing** if $a_n > a_{n+1}$ for all $n \geq 1$.

7. **Definition:** A sequence $\{a_n\}$ is **monotonic** if it is either increasing or decreasing. It is **strictly monotonic** if it is either strictly increasing or strictly decreasing.

8. **Definition:** A sequence $\{a_n\}$ is **bounded above** if there is a real number M such that $a_n \leq M$ for all $n \geq 1$.

9. **Definition:** A sequence $\{a_n\}$ is **bounded below** if there is a real number M such that $a_n \geq M$ for all $n \geq 1$.

10. **Definition:** A sequence $\{a_n\}$ is **bounded** if it is bounded above and bounded below.

11. **Theorem:** Let $\{a_n\}$ be a given sequence, and suppose that f is a function such that $f(n) = a_n$ for all $n \in \mathbb{Z}^+$. If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} a_n = L$.

12. **Theorem:** If $\lim_{n\to\infty} a_n = L$ and f is continuous at L, then $\lim_{n\to\infty} f(a_n) = f(L)$.

13. **Theorem:** If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

14. **Theorem:** $\lim_{n\to\infty} r^n = \begin{cases} 0 & |r| < 1 \\ 1 & r = 1 \\ \text{DNE} & \text{otherwise} \end{cases}$

15. **Theorem (Monotone Sequence Theorem):** Every bounded monotonic sequence converges.

16. Examples: 8.1, p. 434: 5-8, 9, 11, 13, 20, 21, 23, 33, 41, 43

Next Time

1. 8.1: Sequences (Cont.)

2. **Turn in** 8.1 WeBWorK 02: 1, 6