MATH 152/153

Today

1. WeBWorK/Questions
2. 8.3 The Integral and Comparison Tests

Goals:

1. 8.8.3 The Integral and Comparison Tests (Understand the use of these convergence tests)

Where is today's material used?

1. Series appear frequently in chemistry and physics as a means of approximating functions.

8.3 The Integral and Comparison Tests

1. **Theorem (The Integral Test):** Let f be continuous, positive, and decreasing on $[1, \infty)$, and let $a_n = f(n)$ for $n \in \mathbb{Z}^+$. Then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)dx$ converges.

2. **Theorem (Comparison Test):** Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series with $a_n, b_n > 0$ for all n.

   (a) If $\sum_{n=1}^{\infty} b_n$ converges and $a_n \leq b_n$ for all $n \in \mathbb{Z}^+$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) If $\sum_{n=1}^{\infty} b_n$ diverges and $a_n \geq b_n$ for all $n \in \mathbb{Z}^+$, then $\sum_{n=1}^{\infty} a_n$ diverges.

3. **NOTE:** The Integral and Comparison Tests still apply if "for all $n \in \mathbb{Z}^+$" is replaced by "for all n sufficiently large."

4. **Theorem (Limit Comparison Test):** Let $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ be series with $a_n, b_n > 0$ for all n. If $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c$ with $0 < c < \infty$, then both series converge or both series diverge.

Next Time

1. 8.3.5 The Ratio Test