MATH 153

Today

1. WeBWorK/Questions
2. 8.4 Alternating Series, Absolute Convergence, and the Ratio Test

Goals:

1. 8.4 Alternating Series, Absolute Convergence, and the Ratio Test (Understand the definition of alternating series, the definition and significance of absolute convergence, and how and when to apply the Ratio Test to determine convergence of a series)

Where is today's material used?

1. Series appear frequently in chemistry and physics as a means of approximating functions.

8.4 Alternating Series, Absolute Convergence, and the Ratio Test

1. **Definition:** An **alterating series** is a series whose terms alternate in sign: $\sum_{n=0}^{\infty}(-1)^n b_n$ or $\sum_{n=0}^{\infty}(-1)^{n-1} b_n$ with $b_n > 0$.
2. **Theorem (Alternating Series Test):** If the alternating series $\sum_{n=0}^{\infty}(-1)^n b_n$ satisfies (i) $b_{n+1} \leq b_n$ for all n and (ii) $\lim_{n\to\infty} b_n = 0$, then the series converges.
3. **Theorem:** If an alternating series converges, then the error in the nth partial sum is no larger than the first neglected term.

4. **Definition:** A series $\sum_{n=1}^{\infty} a_n$ is **absolutely convergent** if $\sum_{n=1}^{\infty} |a_n|$ converges. It is **conditionally convergent** if it is convergent but not absolutely convergent.

5. **Theorem:** If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then it is convergent.

6. **Theorem (Ratio Test):** Assume $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$ (allowing $L = \infty$).

   (a) If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

   (b) If $L > 1$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

   (c) If $L = 1$, then the test is inconclusive.

7. Examples: 8.4, p. 463: 3-8, 9-12, 13-16, 19-40

Next Time

1. 8.4 The Ratio Test and the Root Test
2. Exam review
3. **Turn in** 8.4 WeBWorK 09: 2, 5