Do Together

Put sheet on Elmo

NOTE: the first finitely many terms don't matter!

Convergence tests: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series.

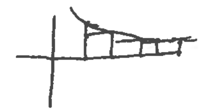
Test for Divergence. 1) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

Algebraic properties 2) If $\sum_{n=1}^{\infty} a_n$ & $\sum_{n=1}^{\infty} b_n$ are convergent, & c then so are $\sum_{n=1}^{\infty} c a_n = c\sum_{n=1}^{\infty} a_n$

$\sum_{n=1}^{\infty} a_n \pm b_n = \sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{\infty} b_n$

Geometric series $\sum_{n=1}^{\infty} ar^n$ cvgs iff $|r|<1$.

Integral Test 3) $\sum_{n=1}^{\infty} a_n$ is convergent iff $\int_1^{\infty} f(x)dx$ is cvgt, where $f(x)$ is a continuous pos, dec. fcn on $[1,\infty)$ & $f(n)=a_n$.



p-series 4) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is cvgt iff $p>1$.

Comparison test 5) Sps $a_n, b_n \geq 0$ $\forall n$,

(a) If $\sum_{n=1}^{\infty} b_n$ is cvgt & $a_n \leq b_n$ $\forall n$, then $\sum a_n$ is cvgt.

(b) if $\sum_{n=1}^{\infty} b_n$ is div. & $a_n \geq b_n$ $\forall n$, then $\sum a_n$ is div.

Limit comparison test If a_n, b_n are all positive & $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then both series converge or both diverge.

factorials

Alternating series test If $a_n > 0$ $\forall n$, a_n is a decreasing sequence, and $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

factorials

Abs. cvg. vs. conditionally cvg.

abs ($\Rightarrow$ cvgt)

Error: $\leq 1^{st}$ neglected term

Ratio Test: If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then $\sum_{n=1}^{\infty} a_n$ is abs. cvg. factorials!

If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$, div.

If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, NO INFO! [like $f''=0$ for concavity]

pf: compare to cvgt geom series

Root Test: If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then $\sum_{n=1}^{\infty} a_n$ is abs cvgt.

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, then $\sum_{n=1}^{\infty} a_n$ is D

If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, inconclusive.

Examples: p. 784:

2. Looks like $\frac{1}{n}$: use limit comparison? $\lim_{n\to\infty} \frac{\frac{n-1}{n^2+n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2-n}{n^2+n} = 1.$ Crgs
4. alt-series $\left(\frac{n-1}{n^2+n}\right)' = \frac{n^2+n-(2n+1)(n-1)}{(n^2+n)^2} = \frac{n^2+n-2n^2+n+1}{(n^2+n)^2} = \frac{-n^2+2n+1}{(n^2+n)^2} < 0$ for n suff. large. Crgs
6. n^{th} power: try root test!: $\lim_{n\to\infty} \sqrt[n]{\left|\frac{3n}{1+8n}\right|^n} = \lim_{n\to\infty} \frac{3n}{1+8n} = \frac{3}{8} < 1.$ Crgs!
8. Factorial: Try ratio test! $\lim_{k\to\infty} \frac{\frac{2^{k+1}(k+1)!}{(k+3)!}}{\frac{2^k(k!)}{(k+2)!}} = \lim_{k\to\infty} \frac{2(k+1)\cancel{k!}\cancel{(k+2)!}}{\cancel{k!}(k+3)\cancel{!}} = 2 > 1.$ DIV.
10. I see that I can integrate! $\int_1^{\infty} x^2 e^{-x^3}\,dx = -\frac{1}{3}e^{-x^3}\Big|_1^{\infty} = -\frac{1}{3}(0-1) = \frac{1}{3}.$

WS: Book probs, p. 758

765 12.3 HW: 5, ~~7, 9~~, -23 odd
770 12.4 HW: 3-31 odd
775 12.5 HW: 3-19 odd
781 12.6 HW: 3-27 odd
784 12.7 HW: 1-37 odd