MATH 153

Today

1. WeBWorK/Questions
2. 8.7 Taylor and Maclaurin Series

Goals:

1. 8.7 Taylor and Maclaurin Series (Understand how to determine whether a given function has a power series representation)

Where is today's material used?

1. Power series are frequently used to approximate more complicated functions in physics and chemistry.
2. Power series techniques are used to solve certain differential equations.

8.7 Taylor and Maclaurin Series

1. **Theorem:** If f has the power series representation $f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n$ about a for $|x - a| < R$, then $c_n = \frac{f^{(n)}(a)}{n!}$.
2. **Definition:** The **Taylor series expansion of f about a** is the series for f in the theorem. If $a = 0$, we call it the **Maclaurin series for f**.
3. **Definition:** The **nth-degree Taylor polynomial of f at a** is the polynomial $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x - a)^k$. The nth-order remainder is $R_n = f(x) - T_n(x)$.
4. **Theorem (Taylor's Formula):** Suppose that f has $n + 1$ derivatives on an interval I containing a. Then if $x \in I$, there is some z between x

and a such that $R_n(x) = \dfrac{f^{n+1}(z)}{(n+1)!}(x-a)^{n+1}$. (Note that z depends on x.)

5. **Theorem:** If $f(x) = T_n(x) + R_n(x)$ and $\lim_{n\to\infty} R_n(x) = 0$ for $|x-a| < R$, then $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ on $|x-a| < R$.

6. Examples: 8.7, p. 488: 5-8, 11-18, 27-34, 41, 44, 56, 57, 59-62

Next Time

1. 8.8: Applications of Taylor Series

2. **Turn in** 8.7 WeBWorK Set 13: 3, 5