

# MATH 153

## Today

1. WeBWorK/Questions
2. Fourier Series

### Goals:

1. Fourier Series (Understand how to model periodic phenomena with Fourier series)

## Where is today's material used?

1. Fourier series are used to model periodic phenomena in physics and chemistry
2. Fourier methods can also be used in signal processing to remove "noise" from images and sound.

## Fourier Series

1. **Theorem:** Let  $m, n \in \mathbb{Z}$ . Then

$$(a) \int_{-\pi}^{\pi} \sin(nx) \cos(mx) dx = 0$$

$$(b) \int_{-\pi}^{\pi} \sin(nx) \sin(mx) dx = 0 \text{ for } m \neq n$$

$$(c) \int_{-\pi}^{\pi} \cos(nx) \cos(mx) dx = 0 \text{ for } m \neq n$$

2. **Theorem:** Suppose that  $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$  (this is the **Fourier series** of  $f$ ). Then  $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$  and  $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$  for  $n > 0$ , and  $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$ .

3. **Definition:** The  $n = k$  term of a Fourier series is called its ***k*th harmonic**.  $A_k = \sqrt{a_k^2 + b_k^2}$  is the **amplitude** of the  $k$ th harmonic. ( $A_0 = \sqrt{2}a_0$ .)

4. **Definition:** The **energy** of a periodic function  $f$  with period  $2\pi$  is

$$E = \frac{1}{\pi} \int_{-\pi}^{\pi} (f(x))^2 dx.$$

5. **Theorem:**  $E = \sum_{k=0}^{\infty} A_k^2 = A_0^2 + A_1^2 + A_2^2 + \dots$

6. Examples: Handout, p. 540: 5, 6, 15, 16

## Next Time

1. Fourier series (not in book)
2. **Turn in** WeBWorK Set 16: 1, 7