

MTH 249

Exam 1

Tuesday, September 21, 2004

Name: _____

Remember to **show your work**. Unsupported solutions will receive **no credit**.

1. (24 points) Let $u = \langle 3, 1, 2 \rangle$, $v = \langle 1, 5, -2 \rangle$, $w = \langle 4, -1, 1 \rangle$, $x = \langle 7, 2 \rangle$, and $y = \langle -2, 1 \rangle$. Compute any quantity that is meaningful. If it is not meaningful, explain why not.

(a) $u + v$

(b) $v \cdot w$

(c) $3x$

(d) $y - 2u$

(e) $v \times y$

(f) $w \times u$

(g) $(u \cdot v) \cdot w$

(h) $u \times (w \cdot v)$

2. (6 points) Find a unit vector in the direction of $u = \langle 3, -4, 3 \rangle$.

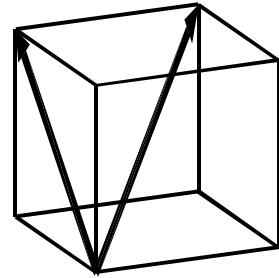
3. (10 points)

(a) (4 points) Find an equation of the sphere centered at $(3, 1, -1)$ with radius 4.

(b) (6 points) Show that the equation $x^2 + y^2 + z^2 = 4x - 2y + 6z + 1$ defines a sphere. What are the center and radius of the sphere?

4. (8 points) Recall that the torque τ about point P generated by a force F applied at a point Q is given by $\tau = \vec{PQ} \times F$. Find the torque generated by a force $F = \langle 3, 1, 6 \rangle$ if $\vec{PQ} = \langle 2, 6, 3 \rangle$.

5. (10 points) What is the angle between the main diagonal of a cube and a face diagonal? (See the figure.)



6. (10 points) \vec{PQ} , \vec{PR} , and \vec{PS} are adjacent edges of a parallelepiped. If the points are $P(-2, 3, 1)$, $Q(2, 3, 6)$, $R(1, -1, 0)$, and $S(1, 3, 1)$, what is the volume of the parallelepiped?

7. (12 points) What are the scalar and vector projections of u onto v if $u = \langle 4, 1, -5 \rangle$ and $v = \langle 2, 6, 1 \rangle$?

8. (6 points) Draw vectors u and v so that $u \times v$ is directed **into** the page.
9. (14 points) A tightrope walker walks one-third of the way across a 60-foot tightrope and pauses. She weighs 135 pounds and deflects the tightrope down by 1.5 feet at this point. What is the magnitude of the tension in each end of the tightrope?
10. **Bonus!** (5 points) Show that if u and v are parallel, then the projection of u onto v is $\pm u$.