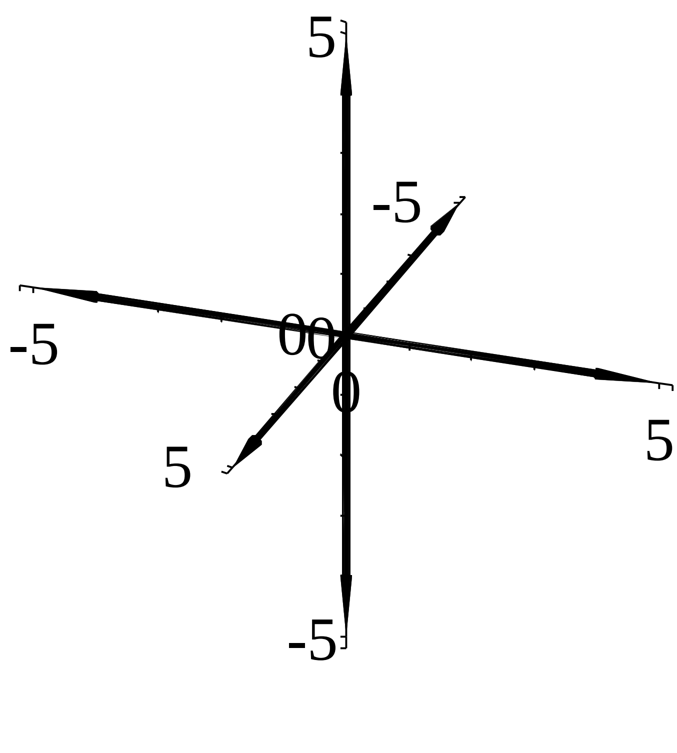
Math 249 Exam II

Tuesday, October 19, 2004

Remember to **show all work**. Unsupported solutions will receive **no credit**.

1. Let $r(t) = < 3t, 3t^2, 2t^3 >$. Most of the exam will refer to this function. Answer the following questions about $r(t)$. Please **label clearly** which part you are working on. You do not have to answer the questions in the order they are asked, but show me which you are answering. Be as neat as possible so I can find your work; that is how you earn partial credit.
   I recommend that you read through all of the questions about $r(t)$ before you begin answering them.
   (a) (5 points) This curve lies on some familiar surface. Identify the surface.
   (b) (5 points) Sketch the surface from (a) as accurately as possible on the axes provided.
   (c) (5 points) Sketch a graph of the curve defined by $r(t)$ on the same set of axes as the surface.
   (d) (5 points) At $r(1)$ on your sketch, draw in T, N, and B. (You do not need to compute them first; just show me what they should look like.)
   (e) (6 points) Find $r'(t)$ and $r''(t)$.
   (f) (6 points) Find parametric equations of the tangent line to the graph of $r(t)$ at $t = 1$.
   (g) (18 points) Compute $T(t), N(t)$, and $B(t)$.
   (h) (6 points) Compute $\kappa(t)$.
   (i) (12 points) Find equations of the normal plane and the osculating plane at $t = 1$. Clearly label which is which!
   (j) (10 points) Find the arc length $s(t)$ measured in the direction of increasing t from $t = 0$. What is $s(1)$? Illustrate $s(1)$ on your graph.
   (k) (12 points) Suppose that $r(t)$ represents the position of a particle at time t. Find its normal and tangential components of acceleration.



2. (10 points) Match each equation with a surface and a set of level curves. The level curves are for $z = -2, -1, 0, 1, 2$ wherever possible.

(a) $x^2 + y^2 - z^2 = 0$

(b) $-x^2 + y^2 + z = 1$

(c) $4x^2 + y^2 + z^2 = 4$

(d) $f(x, y) = \sqrt{x^2 - y^2 - 1}$

(e) $f(x, y) = \sin x + \sin y$

(f) $f(x, y) = \sin(x + y)$