Math 249 Exam III

Tuesday, November 16, 2004

Remember to **show all work**. Unsupported solutions will receive **no credit**.

1. (20 points) Consider $f(x,y) = e^{xy}$.
   (a) Find $\nabla f(x,y)$.
   (b) Sketch the level curves for $z = e^{-1}, 1, e$. Sketch as accurately as possible and label each.
   (c) Draw the gradient at $(1,1)$ on your level curves.
   (d) What does $\nabla f(1,1)$ mean? (I'm not looking for a name; I'm looking for an interpretation!)
   (e) Find the directional derivative of f in the direction of $< -3, 4 >$ at $(1,1)$. How does this number compare with the length of $\nabla f(1,1)$? Why?
2. (10 points) As a car climbs a mountain, a scientist is measuring atmospheric pressure. She determines that the pressure at (x,y) (coordinates on a map) is given by $P(x,y) = 70 + 3\sqrt{x^2+y^2}$ kPa. If $x(t) = (10-t)\cos(2\pi t)$ and $y(t) = (10-t)\sin(2\pi t)$, where t is measured in hours, find $\dfrac{dP}{dt}$ at $t = 2$ hours.
3. (10 points) A toy manufacturer has a mold to create a plastic "Rapunzel's Tower," which is a cone placed on top of a cylinder. The inner radius of each is 5cm, the inside heights of the cylinder and cone are each 15 cm. The cylinder has a bottom. If the plastic is 0.15 cm thick, use differentials to estimate the amount of plastic in each Rapunzel's Tower.
4. (20 points) Find the critical points of $f(x,y) = 2x^2 - 4xy + y^3 + 2$ and classify each as a local maximum, local minimum, or neither.
5. (10 points) Compute $\iint\limits_D xe^{xy}dA$ if $D = [0,1] \times [-1,1]$.
6. (10 points) Compute $\iint\limits_D \dfrac{1}{\sqrt{4-x^2-y^2}}dA$, where $D = \{(x,y) | x \geq 0, y \geq 0, \text{ and } 1 \leq x^2+y^2 \leq 4\}$.
7. (10 points) Compute $\displaystyle\int_0^1 \int_{2x}^2 e^{-y^2} dydx$ by reversing the order of integration. Sketch the region of integration.
8. (10 points) True or False.
   (a) _____ If L is the linearization of f at (x_0, y_0), then the graph of L is the tangent plane to the graph of f at (x_0, y_0).
   (b) _____ For any function f of two variables, $f_{xy} = f_{yx}$.
   (c) _____ If f is continuous on the bounded set D, then f has an absolute maximum and minimum on D.
   (d) _____ If f is continuous on $[a,b] \times [c,d]$, then $\displaystyle\int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy$.
   (e) _____ If f is a function of two variables, then $\nabla f(x_0, y_0)$ is perpendicular the tangent plane to the graph of f at (x_0, y_0).