Math 249 Exam III

Friday, April 1, 2005

Remember to **show all work**. Unsupported solutions will receive **no credit**.

1. (20 points) Consider $f(x,y) = x^2 + 4y^2$.
   (a) Find $\nabla f(x,y)$ and $\nabla f(2,\sqrt{3})$.
   (b) Sketch the level curves for $z = 1, 4, 16$. Sketch as accurately as possible and label each.
   (c) Draw the gradient at $(2,\sqrt{3})$ on your level curves.
   (d) Find the directional derivative of f in the direction of $< -5, 12 >$ at $(2,\sqrt{3})$.
   (e) Explain what the answer to (d) means.
2. (10 points) If $f(x,y) = x^2y^4$, $x(s,t) = 6st - 2$, and $y(s,t) = \sin(s+t)$, find $\dfrac{\partial f}{\partial t}$.
3. (10 points) Use **differentials** to estimate the amount of aluminum needed to make a cylindrical soda can that is 12.22 cm high and has a diameter of 6.35 cm. The aluminum is 0.007 cm thick. (No credit for any other method!)
4. (20 points) Find the absolute maximum and minimum of $f(x,y) = xy^2 - x$ on $\{(x,y) | y \geq 0, x^2 + y^2 \leq 4\}$.
5. (10 points) Compute $\iint\limits_D 2x\cos(x^2+y)dA$ if $D = [0,\sqrt{\pi}] \times [0,\pi]$.
6. (10 points) Compute $\iint\limits_D ydA$, where D is the triangle with vertices $(0,0), (2,2)$, and $(0,4)$.
7. (10 points) Rewrite $\displaystyle\int_0^2 \int_{x^2}^4 f(x,y)dydx$ by reversing the order of integration. Also sketch the region of integration.
8. (10 points) Find an equation of the tangent plane to the graph of $f(x,y) = \dfrac{y}{x}$ at the point $(1,2,2)$.