Math 249 Exam IV

Friday, April 22, 2005

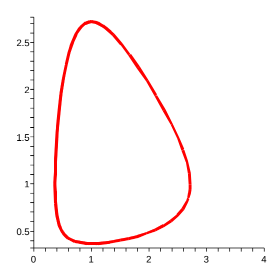
Remember to **show all work**. Unsupported solutions will receive **no credit**.

1. (10 points) Compute $\iint\limits_D \frac{1}{\sqrt{4-x^2-y^2}}dA$, where $D = \{(x,y)|x \geq 0, y \leq 0, \text{ and } 1 \leq x^2+y^2 \leq 4\}$.

2. (8 points) Set up, but **do not** evaluate, an integral giving the surface area of the surface given by $f(x,y) = x^2 - y^2 + 4$ over the region $D = \{(x,y)|x \geq 0, y \geq 0, y \leq 1-x\}$.

3. (10 points) The radius of the sun is about $700,000,000$ meters. The density of the sun at ρ meters from the center is approximately $e^{-0.00000001437\rho+12.365}$ kilograms per cubic meter. What is the mass of the sun? (You may perform your computations on your calculator, but I need to see what it is you are computing!) (Also, if you want to actually see the graph of this function, I recommend setting your y-max to 1×10^{21}.)

4. (32 points) Let $\vec{F}(x,y) =< 2x+y^2, 2xy >$.
   - (a) Sketch a graph of $\vec{F}$ using six points.
   - (b) Convince me that $\vec{F}$ is conservative and find a potential function for $\vec{F}$.
   - (c) Compute $\int\limits_C \vec{F} \cdot d\vec{r}$, where C is the curve parameterized by $r(t) =< t^3e^{t^2}, 1-(t-1)\sin(t)\cos(t) >$ on $[0,1]$.
   - (d) Compute $\int\limits_C \vec{F} \cdot d\vec{r}$, where C is the curve shown.



For 4(d)

5. (20 points) Let $\vec{F} =< 3x+4y, 2x-3y >$, and let C be the unit circle.
   - (a) Evaluate $\int\limits_C \vec{F} \cdot d\vec{r}$ by parameterizing the unit circle.
   - (b) Evaluate $\int\limits_C \vec{F} \cdot d\vec{r}$ by applying Green's Theorem.

6. (8 points) Let $x = u^2 + v, y = u - v^2$. Compute the Jacobian of this transformation. Then (briefly!) describe what we use the Jacobian for.

7. (12 points) Draw each object, if possible. If it is not possible, explain why not.
   - (a) A simple curve that is not closed.
   - (b) A closed curve that is not simple.
   - (c) A simple closed curve.
   - (d) A region that is connected but not simply connected.
   - (e) A region that is simply connected but not connected.
   - (f) An open region that is not connected.