MATH 249

Solutions to Exam 1 **Wednesday, September 18, 2013**

Name:________________________

Remember to **show your work**. Unsupported solutions will receive **no credit**.

Please leave the upper left-hand corner of each page blank so you do not staple over your work. Please write on only one side of the paper.

1. (5 points) What is the distance between the points $(2, 4, 3)$ and $(1, 6, 2)$?

   Solution: $d = \sqrt{(1-2)^2 + (6-4)^2 + (2-3)^2} = \sqrt{6}$.

2. (30 points) Let $u =< 3, -1, 2 >, v =< 4, 1, 5 >, w =< 1, 5, 1 >, x =< -2, 4 >$, and $y =< -1, 3 >$. Compute any quantity that is meaningful. If it is not meaningful, **briefly** explain why not.

(a) $u - 2v$	(b) $w + x$	(c) $x \times y$	(d) $u \times (u \cdot v)$	(e) $u \cdot w$	(f) $\lvert x\rvert$
$=< -5, -3, -8 >$	Wrong dimensions	Wrong dimensions	Can't cross scalar	0	$\sqrt{20}$

3. (15 points) Let $u =< 3, -1, 2 >$ and $v =< 4, 1, 5 >$ Find each of the following.

   (a) The angle between u and v.

   Solution: $\cos\theta = \frac{u \cdot v}{|u||v|} = \frac{21}{\sqrt{14 \cdot 42}} \implies \theta = \pi/6$, or 30 degrees.

   (b) The scalar and vector projections of u onto v.

   Solution: $\text{comp}_v u = \frac{u \cdot v}{|v|} = \frac{21}{\sqrt{42}}$ and $\text{proj}_v u = \frac{21}{42} v = \frac{1}{2} < 4, 1, 5 >$.

   (c) A third vector orthogonal to both u and v.

   Solution: $u \times v =< -7, -7, 7 >$.

4. (10 points) Find an equation of the plane parallel to the plane $3x - 2y + z = 5$ and containing the point $(2, 4, 5)$.

   Solution: $\vec{n} =< 3, -2, 1 >$, so we get $3(x - 2) - 2(y - 4) + 1(z - 5) = 0$.

5. (20 points) Consider the vector-valued function $r(t) =< t^2, 4t, \sin(\pi t/2) >$.

   (a) (4 points) Find $r'(t)$.

   Solution: $r'(t) =< 2t, 4, (\pi/2)\cos(\pi t/2) >$.

   (b) (4 points) Find $\int r(t)dt$.

   Solution: $\int r(t)dt =< t^3/3, 2t^2, -(2/\pi)\cos(\pi t/2) > +\vec{C}$.

   (c) (4 points) Find an equation of some surface on which the space curve given by r lies.

   Solution: The simplest one I see is $x = (y/4)^2$ since $x = t^2$ and $t = y/4$.

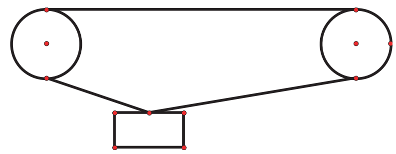
   (d) (8 points) Find parametric equations of the tangent line to $r(t)$ at $t = 1$.

   Solution: We already have $r'(t)$; $r'(1) =< 2, 4, 0 >$. Also, $r(1) =< 1, 4, 1 >$. We get $x = 1 + 2t, y = 4 + 4t, z = 1$.

6. (10 points) A worker pushes a box up a ramp with a horizontal force of 15 Newtons. If the ramp is inclined at 24 degrees to the horizontal, what is the work done in pushing the box 20 meters?

   Solution: $W = \vec{F} \cdot \vec{D} = |vecF||\vec{D}|\cos(\theta) = 15 \cdot 20\cos(24°) = 274.06$N.

7. (10 points) Set up but **do not solve** a system of equations for the following situation: Two workers are moving a 40-pound box on a rope and pulley system across a canyon. They rest when the box is nearly across. At this point, the angle up to the pulley on the left is 26° and the angle up to the pulley on the right is 18°. What is the magnitude of the tension in each end of the rope? **You do NOT need to solve the system.**



Solution: We need $T_1 \sin 26 + T_2 \sin 28 = 40$ (vertical) and $T_1 \cos 26 = T_2 \cos 18$ (horizontal), where T_1 is the magnitude of the tension on the left and T_2 is the magnitude of the tension on the right.

8. (10 points) **BONUS!!!** Show that every rhombus has perpendicular diagonals. (Recall that a rhombus is a parallelogram with all four sides the same length.)