MATH 249

Exam 2 **Wednesday, October 16, 2013**

Name:_____

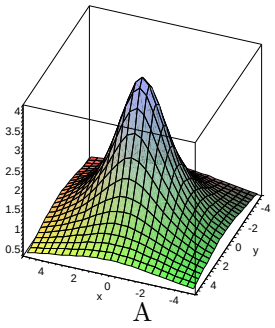
Remember to **show your work**. Unsupported solutions will receive **no credit**.

Please leave the upper left-hand corner of each page blank so you do not staple over your work. Please write on only one side of the paper. You may use Maple as needed, but be sure to tell me when you do.

1. (10 points) Let $r(t) =< 8\cos t, 15\cos t, 17\sin t >$. Compute $T(t)$ and the curvature of r.
2. (25 points) Let $f(x, y) = x^2y + xy^2 + xy$.
   (a) (10 points) Find the linearization of f at the point $(2, 1)$.
   (b) (15 points) Find all extrema and saddle points of f. A graphical argument will not suffice; I need to see your analysis. [Hint: there are four critical points.]
3. (10 points) Consider $f(x, y) = x^2 - y$.
   (a) Find the directional derivative of f in the direction of $< -3, 4 >$ at $(1, 2)$.
   (b) Determine the maximum rate of increase of f at the point $(1, 2)$.
4. (10 points) The surface S is defined by $x^2 - y^2 + 6z^2 = 14$. Find an equation of the tangent plane to S at $(3, 1, 1)$.
5. (10 points) If $f(x, y) = e^{2x+3y}$, $x(s, t) = 4s^2t$, and $y(s, t) = 2st^2$, find $\dfrac{\partial f}{\partial s}$. It is not necessary to simplify nor to express your final answer in terms of s and t.
6. (15 points) Use **differentials** to estimate the amount of aluminum needed to make a cylindrical soda can that is 12.2 cm high and has a diameter of 6.0 cm. The aluminum is 0.05 cm thick. **(No credit for any other method!)** You do not need to simplify your answer.
7. (10 points) True or False/fill-in.
   (a) _____ If u is a vector, then the directional derivative of f in the direction of u is $\nabla f \cdot u$.
   (b) _____ The graph of the linearization of a differentiable function at a point is the tangent plane to the graph of the function at that point.
   (c) _____ The gradient of a function is perpendicular to the graph of the function.
   (d) _____ The gradient of a function is tangent to the graph of the function.
   (e) _____ If $z = f(x, y)$ is differentiable, then $\Delta z = dz$.

Be sure to turn the paper over and answer the matching question on the back!

8. (10 points) Match the letter of each equation with a surface and a set of level curves. The level curves are drawn for at most five values of $f(x, y)$ and correspond to equal changes in z for each graph.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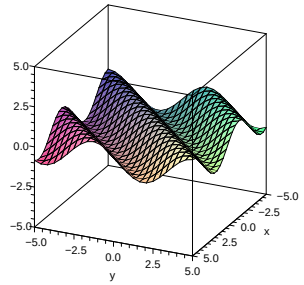
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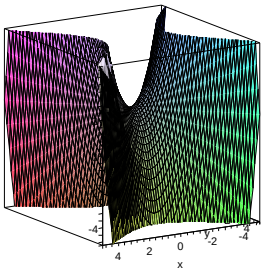
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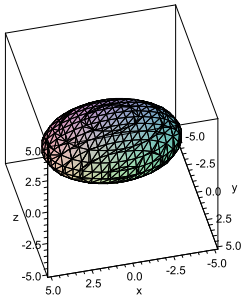
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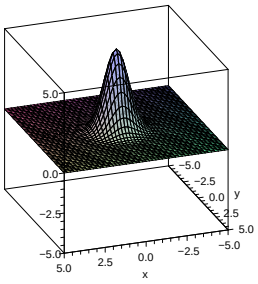
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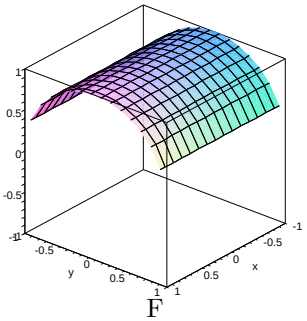
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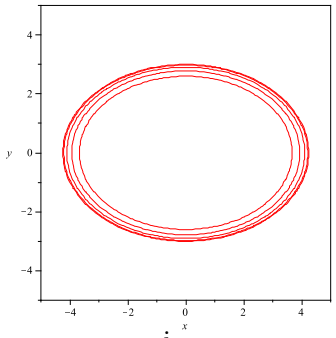
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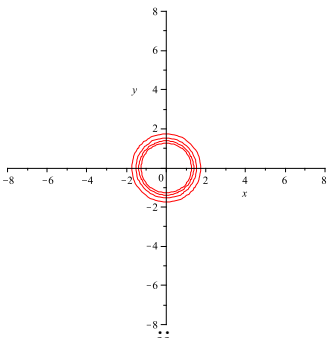
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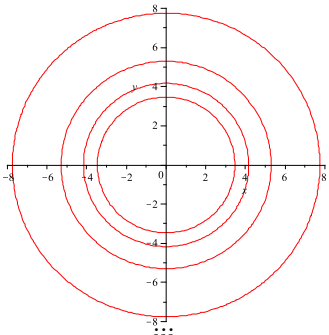
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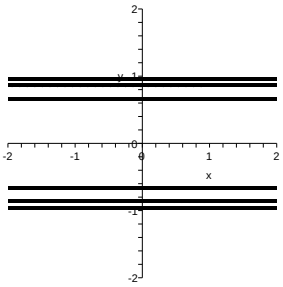
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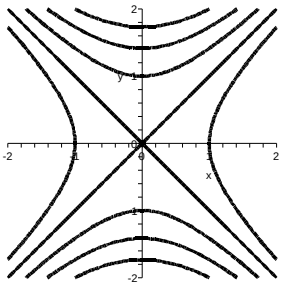
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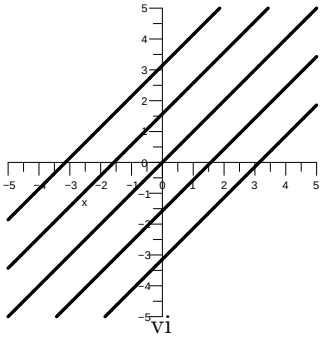
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