

Math 249 Exam III

Wednesday, November 13, 2013

Name: _____

Remember to **show all work**. Unsupported solutions will receive **no credit**.

1. (50 points) Integration fundamentals. For each “set up” problem, use the coordinate system in which you would carry out the integration. (Your score depends on your choice!) For example, if spherical coordinates are more natural, set up the integral with spherical coordinates. You do not need to simplify your integrands.

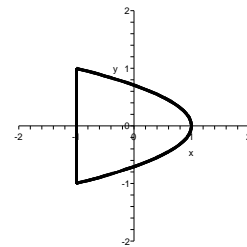
- (a) Compute $\iint_D 3x^2 \cos(2y) dA$ if $D = [1, 3] \times [0, \pi/4]$. **Integrate!** This is the one where I want to see the integration steps, so don't use Maple on this one (except to check).
- (b) Set up $\iint_D x^2 + y^2 dA$, where D is the triangle with vertices $(-1, 1)$, $(3, 1)$, and $(-1, 2)$. **Do not integrate!**
- (c) Set up $\iint_D (x\sqrt{4+x^2+y^2})^3 dA$, where $D = \{(x, y) | 4 \leq x^2 + y^2 \leq 16, x \geq 0, \text{ and } y \geq x\}$. **Do not integrate!**
- (d) Set up an integral to find the mass of a wire with density function x^2y^2 if the wire is in the shape of a circle of radius 4.
- (e) Set up $\iiint_E e^{x^2+y^2+z^2} dV$, where E is the region inside the cylinder $x^2 + y^2 = 4$, above the plane $z = 0$, and below the plane $z = y$. **Do not integrate!**
- (f) Set up a triple integral to find the mass of the solid in the first octant bounded by the graphs of $z^2 = x^2 + y^2$, $x^2 + y^2 + z^2 = 100$, $z = 0$, $y = 0$, and $y = x$, where the density of the solid is $f(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$. **Do not integrate!**

2. (20 points) Consider the vector field $\vec{F} = \langle xz, y, z \rangle$.

- (a) (10 points) Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the line segment joining $(0, 0, 0)$ to $(1, 2, 3)$. **Integrate!** (You may use Maple to compute the integral.)
- (b) (5 points) Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is given by $\vec{r}(t) = \langle t^2, 2t^3, 3t \rangle$ on $[0, 1]$. **Integrate!** (You may use Maple to compute the integral.)
- (c) (5 points) Is \vec{F} conservative? Why or why not?

3. (20 points) Let $\vec{F}(x, y) = \langle 2x + y^2, 2xy \rangle$.

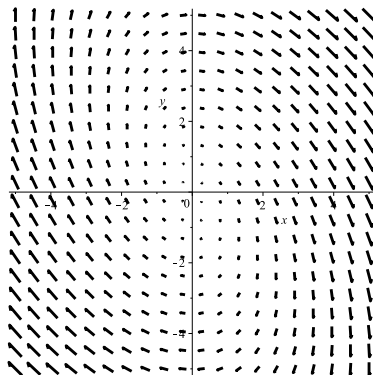
- (a) (10 points) Convince me that \vec{F} is conservative and find a potential function for \vec{F} .
- (b) (5 points) Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve parameterized by $r(t) = \langle t^3 \sin(\pi t/2), 1 - (t - 1) \sin(t) \cos(t) \rangle$ on $[0, 1]$.
- (c) (5 points) Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve shown.



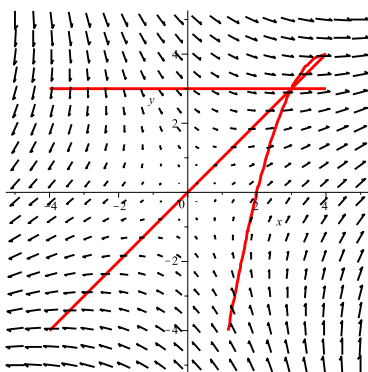
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4. (5 points) Determine (with justification) whether the given vector field is conservative.



5. (5 points) For each of the three given curves in the plane, determine whether $\int_C \vec{F} \cdot d\vec{r}$ is positive, negative, or zero. All curves move left to right.



6. (10 points) **BONUS!!!** Describe a method for determining the surface area of a surface S given by $z = f(x, y)$ over a rectangle R . I am looking for an analysis that leads to a double integral (in the manner we have done seven times so far this semester!). You might not find an exact formula, but do your best to describe the process.