Review for Final Exam

MATH 249

The following is offered without warranty, expressed or implied. You are responsible for all material covered.

Logistics

1. Python and CalcPlot will be available, but you may not access the internet otherwise.
2. You may use our Handy Python Info sheet.
3. No cell phone calculators.
4. You will need to state Green's Theorem, Stokes' Theorem, and the Divergence Theorem without the assistance of your green book.
5. You may use your green book after you hand in the first page.
6. The exam will include roughly half new material and half old material.

Content

1. 3-D coordinates and vectors.
   (a) Dot product, cross product, and their properties and uses (angles between vectors, projections, etc.)
2. Equations of lines and planes
3. Arc length, curvature, T, N, B
4. Multivariable functions
   (a) Graphs
   (b) Partial derivatives
   (c) Equations of tangent planes
   (d) Linearization and differentials
   (e) The chain rule
   (f) Directional derivatives
   (g) The gradient
      i. Used to compute directional derivatives
      ii. Normal to level curves/surfaces
      iii. Direction of greatest increase
   (h) Optimization
      i. Local extrema (D)
      ii. Absolute extrema (Critical points and boundary)
      iii. Lagrange multipliers
5. Multiple integration
   (a) Double integrals
      i. Definition/concept
      ii. Calculation (e.g., midpoint rule)
   (b) Iterated integrals
      i. Definition/concept
      ii. Fubini's Theorem
      iii. General regions
      iv. Polar coordinates ($dA = rdrd\theta$)
      v. Computing area with a double integral

vi. General changes of variable

(c) Triple integrals

i. Definition/concept

ii. Fubini's Theorem

iii. Mass, volume

iv. Cylindrical and spherical coordinates ($dV = rdzdrd\theta, dV = \rho^2 \sin(phi)d\rho d\phi d\theta$)

6. Vector Fields

(a) Definition/concept

(b) Gradient vector fields/conservative vector fields (and potential functions)

(c) Line/path integrals of scalar functions and vector fields (including meaning and interpretation)

(d) Fundamental Theorem for line integrals

(e) Independence of path

(f) Green's Theorem

(g) Curl and divergence

(h) Test for conservatism

7. Parametric surfaces

(a) Surface area

(b) Tangent planes

8. Surface integrals

(a) Scalar functions – definition and computation (parametric and $z = g(x, y)$, what is dS?)

(b) Flux integrals – definition and computation (parametric and $z = g(x, y)$, what is $d\vec{S}$?)

(c) Stokes' Theorem

(d) Divergence Theorem

Advice

(a) Sketch the domain of integration for every integration problem.

(b) Remember to include dA and dV as appropriate to the coordinate system you choose.

(c) **Read the instructions.** For example, some integration problems will just tell you to set up an integral and go on. **Don't waste time integrating those!**

(d) Be very solid on the coordinate transformations.

(e) When in doubt, **think about what things mean!** Also do this when not in doubt.