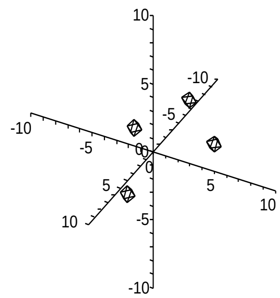
Solutions to Homework Assignment 1

MATH 249

Section Stewart 6e 12.1, Page 769

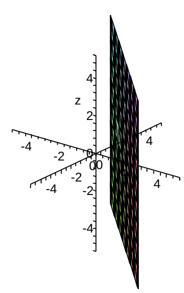
1-9, 11, 15-18, 23-33 odd, 35-38, 41

2.



4. The length of the diagonal is $\sqrt{(2-0)^2+(3-0)^2+(5-0)^2} = \sqrt{38}$ units.

5. We will see in Section 12.5 that this is an equation of a plane, as it appears to be. In fact, it is a vertical plane.



7. $PQ = \sqrt{(-2-1)^2+(4-2)^2+(0-(-1))^2} = \sqrt{14}$. $PR = \sqrt{(-2-(-1))^2+(4-1)^2+(0-2)^2} = \sqrt{14}$. $QR = \sqrt{(1-(-1))^2+(2-1)^2+(-1-2)^2} = \sqrt{14}$. Since all three sides have the same length, the triangle is equilateral.

9. (a) The points are collinear if and only if the distances add up correctly. We have $AB = \sqrt{(5-7)^2+(1-9)^2+(3-(-1))^2} = \sqrt{84} = 2\sqrt{21}$. $AC = \sqrt{(5-1)^2+(1-(-15)^2)+(3-11)^2} = \sqrt{336} = 4\sqrt{21}$. $BC = \sqrt{(7-1)^2+(9-(-15))^2+(-1-11)^2} = \sqrt{756} = 6\sqrt{21}$. Since $AB+AC = BC$, the three points are collinear (and A is between B and C).

24. This is a plane parallel to the yz-plane and through the point $(10, 0, 0)$.

36. $x^2+y^2+z^2 \leq 4, z \geq 0$.

39. In order for a point (x, y, z) to be equidistant from both $(-1, 5, 3)$ and $(6, 2, -2)$, it must satisfy the equation $\sqrt{(x+1)^2+(y-5)^2+(z-3)^2} = \sqrt{(x-6)^2+(y-2)^2+(z+2)^2}$. This leads to

$$\begin{aligned}
(x+1)^2 + (y-5)^2 + (z-3)^2 &= (x-6)^2 + (y-2)^2 + (z+2)^2 \\
(x^2+2x+1) + (y^2-10y+25) + (z^2-6z+9) &= (x^2-12x+36) + (y^2-4y+4) + (z^2+4z+4) \\
2x - 10y - 6z + 35 &= -12x - 4y + 4z + 44 \\
14x - 6y - 10z &= 9.
\end{aligned}$$

This is a plane.