Solutions to Homework Assignment 3

MATH 249

Section Stewart 6e 12.3, Page 784

1-20, 23, 24, 26, 27, 35-40, 45, 47, 53, 58

8. $(4\hat{j} - 3\hat{k}) \cdot (2\hat{i} + 4\hat{j} + 6\hat{k}) = 0(2) + 4(4) + (-3)(6) = -2$.

14. The dot product gives the vendor's total income that day from hamburgers, hot dogs, and soft drinks.

16. $< \sqrt{3}, 1 > \cdot < 0, 5 >= \sqrt{3}(0) + 1(5) = 5$. $|a| = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ and $|b| = \sqrt{0^2 + 5^2} = 5$. Thus $a \cdot b = 5 = (2)(5)\cos\theta$, so $\cos\theta = \dfrac{1}{2}$. Therefore, $\theta = 60$ degrees.

24. (a) Since $u = -\dfrac{3}{4}v$, u and v are parallel.

    (b) $u \cdot v = 1(2) + (-1)(-1) + (2)(1) = 5$. This u and v are neither parallel nor perpendicular.

    (c) $u \cdot v = a(-b) + b(a) + c(0) = 0$, so u and v are perpendicular.

38. The scalar projection of b onto a is $\dfrac{< -2, 3, -6 > \cdot < 5, -1, 4 >}{| < -2, 3, -6 > |} = \dfrac{-37}{7}$. The vector projection of b onto a is therefore $\dfrac{-37}{7}\dfrac{< -2, 3, -6 >}{7} = \dfrac{-37}{49} < -2, 3, -6 >$.

58. (a) The the length of any side of any triangle is less than or equal to the sum of the lengths of the other two sides.

    (b)

$$\begin{aligned} |a+b|^2 &= (a+b)\cdot(a+b) \\ &= a\cdot a + a\cdot b + b\cdot a + b\cdot b \\ &= |a|^2 + 2|a||b|\cos\theta + |b|^2 \\ &\le |a|^2 + 2|a||b| + |b|^2 \\ &= (|a| + |b|)^2. \end{aligned}$$

    Therefore, $|a + b| \le |a| + |b|$.