

Solutions to Homework Assignment 4

MATH 249-01 and -02

Section 12.4, Page 820

1-7, 9-12, 14, 15, 25, 26, 30, 31, 34, 35, 36, 45

2. $\langle 1, -1, 1 \rangle \times \langle 2, 4, 6 \rangle = \langle 1(6) - (-1)(4), -[1(6) - (-1)(2)], 1(4) - (1)(2) \rangle = \langle 10, -8, 2 \rangle$. You can check the orthogonality.
6. $\langle 1, e^t, e^{-t} \rangle \times \langle 2, e^t, -e^{-t} \rangle = \langle e^t(-e^{-t}) - e^{-t}(e^t), -[1(-e^{-t}) - e^{-t}(2)], 1(e^t) - e^t(2) \rangle = \langle -2, 3e^{-t}, -e^t \rangle$.
13. Only (a), (c), and (f) are meaningful. The others are all attempting to take a cross product in which at least one factor is a scalar.
14. $|u \times v| = 5 \cdot 10 \sin(60^\circ) = 25\sqrt{3}$. According to the right-hand rule, it will be directed into the page.
16. (a) Since the z -axis is perpendicular to the xy -plane, we have $|a \times b| = 3(2) \sin(90) = 6$.
(b) The cross product is in the xy -plane, so the z -component is 0. It is also going to be in the second quadrant (using the right-hand rule), so its x -component is positive and its y -component is negative.
18. $\langle 3, 1, 2 \rangle \times (\langle -1, 1, 0 \rangle \times \langle 0, 0, -4 \rangle) = \langle 3, 1, 2 \rangle \times \langle -4, -4, 0 \rangle = \langle 8, -8, -8 \rangle$, while $(\langle 3, 1, 2 \rangle \times \langle -1, 1, 0 \rangle) \times \langle 0, 0, -4 \rangle = \langle -2, -2, 4 \rangle \times \langle 0, 0, -4 \rangle = \langle 8, -8, 0 \rangle$.
30. $\vec{PQ} = \langle -3, 2, -1 \rangle$ and $\vec{PR} = \langle 1, -1, 1 \rangle$. A vector orthogonal to both of these is also orthogonal to the plane through P, Q , and R . Such a vector is $\langle -3, 2, -1 \rangle \times \langle 1, -1, 1 \rangle = \langle 1, 2, 1 \rangle$. The area of the parallelogram the three points defined is the norm of this, or $\sqrt{6}$, so the area of the triangle is half that: $\frac{\sqrt{6}}{2}$.
35. We need vectors representing the edges: $u = \vec{PQ} = \langle 2, 1, 1 \rangle$, $v = \vec{PR} = \langle 1, -1, 2 \rangle$, and $w = \vec{PS} = \langle 0, -2, 3 \rangle$. The volume of the parallelepiped is $|u \cdot (v \times w)| = |\langle 2, 1, 1 \rangle \cdot (\langle 1, -1, 2 \rangle \times \langle 0, -2, 3 \rangle)| = |\langle 2, 1, 1 \rangle \cdot \langle 1, -3, -2 \rangle| = 3$.
39. The magnitude of the torque is $|\tau| = |r \times F| = |r||F| \sin \theta = (0.18)(60) \sin(80) \approx 10.64 \text{ N}\cdot\text{m}$. Note that we had to convert r to meters since the force was given in Newtons.
49. (a) It does not follow! For example, consider $a = \hat{i}$, $b = \hat{j}$, and $c = \hat{k}$.
(b) It does not follow! Consider $a = \langle 1, 0, 0 \rangle$, $b = \langle 1, 0, 0 \rangle$, and $c = \langle 2, 0, 0 \rangle$. Since all three are parallel, cross products will be zero.
(c) Now we're talking! With $a \cdot b = a \cdot c$, we have $a \cdot (b - c) = 0$. Likewise, we have $a \times (b - c) = 0$. Thus, a is both perpendicular to $b - c$ and parallel to $b - c$, so $b - c$ must be the zero vector. Therefore, $b = c$.