# Solutions to Homework Assignment 5

**MATH 249**

**Section Stewart 6e 12.5, Page 802**

**1, 2, 3, 6, 9, 11, 13, 14, 19-24, 26, 30, 37, 43, 44, 49-56, 62, 69-73**

3. A vector equation should look like $r(t) =< 2, 2.4, 3.5 > +t < 1, 2, -2/3 >$ . Comparing components gives us the parametric equations $x = 2 + t, y = 2.4 + 2t, z = 3.5 - 2t/3$.
6. A direction vector is just $< 1, 2, 3 >$, so our equations are $x = t, y = 2t, z = 3t$. Symmetric equations are $x = \frac{y}{2} = \frac{z}{3}$.
9. A direction vector is $< 2 - 0, 1 - 1/2, -3 - 1 >=< 2, 1/2, -4 >$ . Parametric equations are then $x = 2+2t, y = 1+t/2, z = -3-4t$. Solving for $t$ gives the symmetric equations $\frac{x-2}{2} = 2y-2 = \frac{z+3}{-4}$.
11. The given line has parametric equations $x = t - 2, y = 2t, z = t + 3$. Its direction vector is thus $< 1, 2, 1 >$ (from the coefficients of $t$). We get $x = 1 + t, y = -1 + 2t, z = 1 + t$.
19. $L_1$ is in the direction $< -6, 9, -3 >$, and $L_2$ is in the direction $< 2, -3, 1 >$ . These are parallel, so the lines are parallel.
21. $L_1$ is in the direction $< 1, 2, 3 >$ and $L_2$ is in the direction $< -4, -3, 2 >$, so these are not parallel. Do they meet? In $L_1$, we have $x = t$; in $L_2$ we have $x = -4s + 3$. Thus, if the lines meet, they do so when $t = -4s + 3$. At this point in $L_1$, we have $y = 2t + 1 = 2(-4s + 3) + 1 = -8s + 7$. For $L_2$, the $y$-coordinate is $y = -3s + 2$. For these to be the same, we must have $-8s + 7 = -3s + 2$, so $s = 1$. This also gives $t = -1$, so in $L_1$, $z = 3t + 2 = -1$, while in $L_2$, $z = 2s + 1 = 3$. Since these are different, the lines do not meet. Thus, these are skew lines.
23. We have $-2x + y + 5z = 6(-2) + 3(1) + 2(5) = 1$.
26. The direction of the line is $< 1, 2, -3 >$, so our plane has equation $x + 2y - 3z = 1(-2) + 2(8) - 3(10) = -16$.
43. We need $(3 - t) - (2 + t) + 2(5t) = 9$, so $t = 1$. The point is $(2, 3, 5)$.
49. The normals are $< 1, 4, -3 >$ and $< -3, 6, 7 >$ . These are not parallel, so neither are the planes. $< 1, 4, -3 > \cdot < -3, 6, 7 >= 0$, so the planes are perpendicular.
50. The normals are $< 1, -4, 2 >$ and $< 3, -12, 6 >$, so these planes are parallel.
55. We know that the line must be perpendicular to both normals since it lies in both planes, so the line has direction $< 1, 1, 1 > \times < 1, 2, 2 >=< -6, -1, -7 >=< 0, -1, 1 >$ . Now we need a point on the line. Put $z = 0$ in both equations, giving $x + y = 1$ $x + 2y = 1$. Thus $y = 0$ and $x = 1$, so $(1, 0, 0)$ is on the line. Thus we get $x = 1, y = -t, z = t$.
62. (a) We must simultaneously solve $1 + t = 2 - s, 1 - t = s$, and $2t = 2$ (equating corresponding components of the two vectors). The last gives $t = 1$, so $s = 0$.
    (b) Such a plane has a normal perpendicular to both lines: $n =< 1, -1, 2 > \times < -1, 1, 0 >=< -2, -2, 0 >$ . Using $t = 0$ gives the point $< 1, 1, 0 >$ in the plane. Our equation is then $-2x-2y = -2(1) - 2(1)$, or just $x + y = 2$.
69. Applying the formula gives $\frac{|3(1) + 2(-2) + 6(4) - 5|}{\sqrt{9 + 4 + 36}} = \frac{18}{7}$.
73. The distance from $(x, y, z)$ in the first plane to the second plane is $\frac{|ax + by + cz + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.