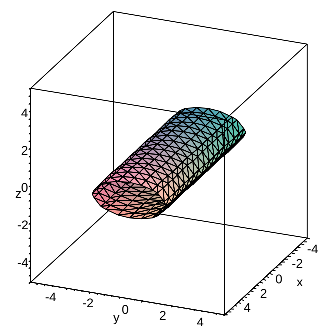
MATH 249-01 and -02

Solutions to Homework Assignment 7

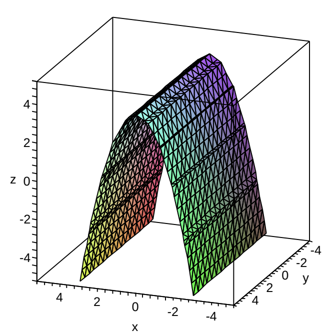
Section 12.6, Page 837

1, 3-8, 11, 14, 16, 17, 18, 21-28, 33, 35, 45

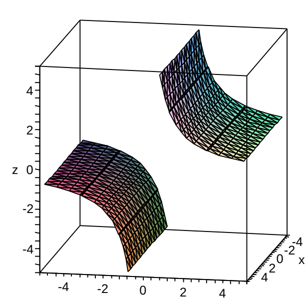
1. (a) It represents a parabola.
   (b) It represents a vertical parabolic cylinder.
   (c) This is also a parabolic cylinder with its vertex along the x-axis.
3. This is independent of x, so it is a cylinder along the x-axis. In each cross-section $x = k$, the trace is an ellipse, so this is an elliptical cylinder.



4. Since this is independent of y, it is a cylinder along the y-axis. In the xz-plane, it is a parabola, so the graph is a parabolic cylinder.



6. This is a hyperbolic cylinder along the x-axis.



16. When z is a constant, the trace is an ellipse. When x or y is constant, the trace is a parabola. This gives an elliptic paraboloid.

17. In each of the planes $x = k, y = k, z = k$, the trace is an ellipse. This surface is an ellipsoid.

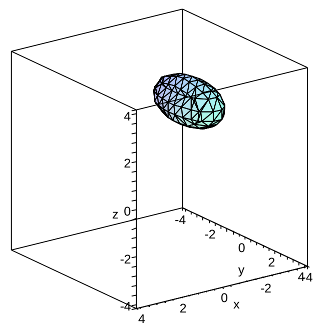
22. This is an ellipsoid. Its longest axis is along the z-axis, so this matches with IV.

24. This is a hyperboloid of two sheets whose axis is the y-axis. It matches with III.

26. This is a cone with elliptical cross sections and the y-axis for its axis; it matches with I.

28. This is a hyperbolic paraboloid. The only one graphed is V.

33. We complete the square: $4x^2 + (y-2)^2 + 4(z-3)^2 = -36 + 4 + 36 = 4$. We then get $x^2 + \frac{(y-2)^2}{2^2} + (z-3)^2 = 1$. This is an ellipsoid whose center is at $(0, 2, 3)$.



45. This is very similar to the focus-directrix definition of a parabola, so I'm thinking it's probably a paraboloid. Let's check it out. The square of the distance from the generic point (x, y, z) to the point $(-1, 0, 0)$ is $(x+1)^2 + y^2 + z^2$. The square of the distance from (x, y, z) to the plane $x = 1$ is $(x-1)^2$. These two quantities are supposed to be equal. We get $x^2 + 2x + 1 + y^2 + z^2 = x^2 - 2x + 1$, so $-4x = y^2 + z^2$. This is indeed a circular paraboloid.