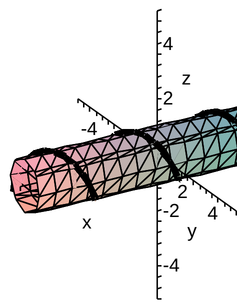
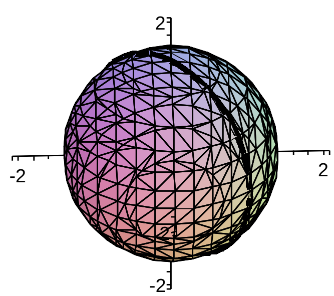
Solutions to Homework Assignment 8

3. $\lim_{t\to 0^+} t\ln t = \lim_{t\to 0^+} \frac{\ln t}{1/t} \overset{\text{H}}{=} \lim_{t\to 0^+} \frac{1/t}{-1/t^2} = \lim_{t\to 0^+}(-t) = 0$. The overall limit is $< 1, 0, 0 >$.

9. Notice that $y^2 + z^2 = 1$, so the curve lies on that cylinder. This will be a helix revolving around the x-axis every π units (instead of 2π because of the $2t$ inside the sine and cosine).



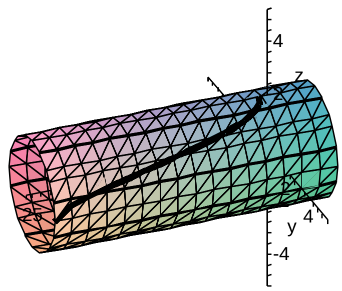
14. We have $x^2 + y^2 + z^2 = 2$, so the curve lies on that sphere. The x- and y-coordinates are equal and oscillate between ± 1, so the graph is on the plane $y = x$. Also, the z-coordinate oscillates between $\pm\sqrt{2}$. We will just get the circle that is the intersection of $y = x$ and the sphere.



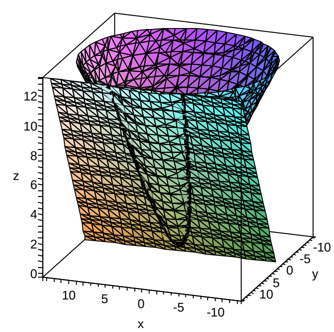
20. Since $y = x^2$, the curve must lie on that parabolic cylinder. At the same time, z is a decaying exponential. This matches II.

22. Since $x^2 + y^2 = z^2$, the curve lies on that cone. It climbs the cone by wrapping around it and falling exponentially; this matches I.

29. Notice that $y^2 = -z^2 + 4$, so $y^2 + z^2 = 4$. The graph will lie on that cylinder.



37. Since $z=\sqrt{x^2+y^2}$ and $z=1+y$, $(1+y)^2=x^2+y^2$. Thus $1+2y+y^2=x^2+y^2$, and $y=\dfrac{1}{2}x^2-\dfrac{1}{2}$. This parabola rises up the top half of the cone $z^2=x^2+y^2$. Choosing x as the parameter, we have $x=t, y=\dfrac{1}{2}t^2-\dfrac{1}{2}$, and $z=\sqrt{t^2+(t^2-1)^2/4}=\sqrt{t^2+\dfrac{t^4-2t^2+1}{2}}=\sqrt{\dfrac{t^4+2t^2+1}{4}}=\dfrac{t^2+1}{2}$. The vector function is $r(t)=<t,(t^2-1)/2,(t^2+1)/2>$.



43. (a) Let $u(t)=<f_1(t),\ldots,f_n(t)>$ and $v(t)=<g_1(t),\ldots,g_n(t)>$. Then

$$\begin{aligned}\lim_{t\to a}(u(t)+v(t)) &= \lim_{t\to a}(<f_1(t),\ldots,f_n(t)>+<g_1(t),\ldots,g_n(t)>)\\ &= \lim_{t\to a}<f_1(t)+g_1(t),\ldots,f_n(t)+g_n(t)>\\ &=<\lim_{t\to a}(f_1(t)+g_1(t)),\ldots,\lim_{t\to a}(f_n(t)+g_n(t))>\\ &=<\lim_{t\to a}f_1(t)+\lim_{t\to a}g_1(t),\ldots,\lim_{t\to a}f_n(t)+\lim_{t\to a}g_n(t)>\\ &=<\lim_{t\to a}f_1(t),\ldots,\lim_{t\to a}f_n(t)>+<\lim_{t\to a}g_1(t),\ldots,\lim_{t\to a}g_n(t)>\\ &= \lim_{t\to a}<f_1(t),\ldots,f_n(t)>+\lim_{t\to a}<g_1(t),\ldots,g_n(t)>\\ &= \lim_{t\to a}u(t)+\lim_{t\to a}v(t).\end{aligned}$$

Note that we needed to know that u and v both have limits as $t\to a$ in order to say that $\lim_{t\to a}(f_i(t)+g_i(t))=\lim_{t\to a}f_i(t)+\lim_{t\to a}g_i(t)$ for each i.

(b) The others are proved similarly.