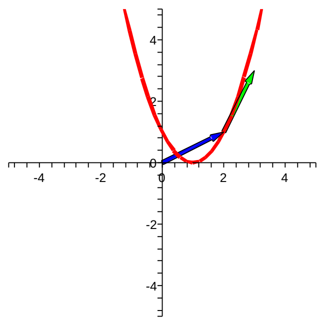
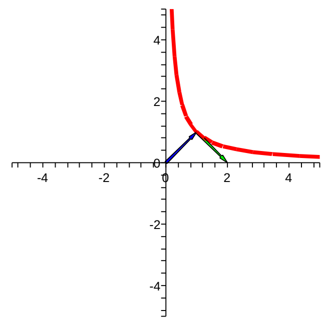
Solutions to Homework Assignment 9

5. The problem was $r(t) =< t+1, t^2 >$. Notice that $y = (x-1)^2$, so this is the parabola $y = x^2$ shifted right 1 unit. $r'(t) =< 1, 2t >$. At $t = 1$, this is $< 1, 2 >$. The graphs are below. The position vector is blue and the tangent vector is green.

6. Here, $y = 1/x$ and neither x nor y can be negative or zero. We just get the arm of the hyperbola in the first quadrant. $r'(t) =< e^t, -e^{-t} >$. At $t = 0$, this is $< 1, -1 >$.



Number 5



Number 6

10. $r'(t) =< \sec^2 t, \sec t \tan t, -2/t^3 >$.

13. $r'(t) = \left\langle 2te^{t^2}, 0, \dfrac{3}{1+3t} \right\rangle$.

15. $r'(t) = \vec{b} + 2t\vec{c}$.

21. $r'(t) =< 1, 2t, 3t^2 >$, so $T(t) = \dfrac{< 1, 2t, 3t^2 >}{\sqrt{1+4t^2+9t^4}}$. Thus, $T(1) = \dfrac{1,2,3}{\sqrt{14}}$. $r''(t) =< 0, 2, 6t >$. $r'(t) \times r''(t) =< 1, 2t, 3t^2 > \times < 0, 2, 6t >=< 12t^2 - 6t^2, -(6t-0), 2-0 >=< 6t^2, -6t, 2 >$.

23. $r'(t) =< 1/\sqrt{t}, 3t^2 - 1, 3t^2 + 1 >$, so $r'(1) =< 1, 2, 4 >$. (Note that $t = 1$ gives the point (3,0,2).) The point on the curve at $t = 1$ is $< 3, 0, 2 >$. Thus, parametric equations of the line are $x = 3 + t, y = 2, z = 4 + 2t$.

33. $\displaystyle\int_0^1 < 16t^3, -9t^2, 25t^4 > dt = < 4t^4, -3t^2, 5t^5 > \Big|_0^1 =< 4, -3, 5 >$.

42. Let $u(t) =< g(t), h(t), k(t) >$. Then

$$\begin{aligned}
\frac{d}{dt}[f(t)u(t)] &= \frac{d}{dt}[f(t) < g(t), h(t), k(t) >] \\
&= \frac{d}{dt} < f(t)g(t), f(t)h(t), f(t)k(t) > \\
&= \left\langle \frac{d}{dt}[f(t)g(t)], \frac{d}{dt}[f(t)h(t)], \frac{d}{dt}[f(t)k(t)] \right\rangle \\
&=< f'(t)g(t) + f(t)g'(t), f'(t)h(t) + f(t)h'(t), f'(t)k(t) + f(t)k'(t) > \\
&=< f'(t)g(t), f'(t)h(t), f'(t)k(t) > + < f(t)g'(t), f(t)h'(t), f(t)k'(t) > \\
&= f'(t) < g(t), h(t), k(t) > + f(t) < g'(t), h'(t), k'(t) > \\
&= f'(t)u(t) + f(t)u'(t).
\end{aligned}$$