Solutions to Homework Assignment 11

MATH 249-01 and -02

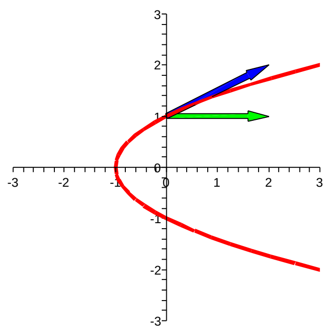
Section 13.4, Page 878

3, 4, 7, 9, 12, 13, 15, 16, 19, 27, 33, 34, 36

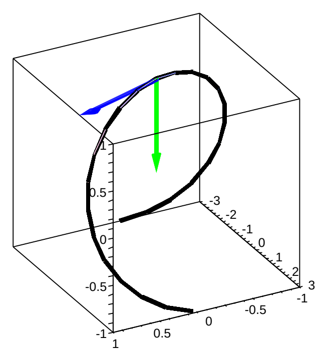
Note: 3 and 7 don't match the book; my vector-valued functions are different.

3. For $r(t) =< t^2, t >$, $v(t) = r'(t) =< 2t, 1 >$. $a(t) = v'(t) =< 2, 0 >$. $|r'(t)| = \sqrt{4t^2+1}$. Notice that $x = y^2 - 1$, so the path is a parabola that opens onto the x-axis and has its vertex at $(-1, 0)$. At $t = 1$ we have $r(1) =< 0, 1 >, v(1) =< 2, 1 >$, and $a(1) =< 2, 0 >$. These are graphed with the curve below. For this and Number 7, the velocity vector is blue and the acceleration vector is green.

7. For $r(t) =< \sin t, t, \cos t >$, $v(t) =< \cos t, 1, -\sin t >, a(t) =< -\sin t, 0, -\cos t >$, and $|v(t)| = \sqrt{2}$. Thus $r(0) =< 0, 0, 1 >, v(0) =< 1, 1, 0 >$, and $a(0) =< 0, 0, -1 >$. The graph, a helix, is below.



Number 3



Number 7

9. $v(t) =< 2t, 3t^2, 2t >, a(t) =< 2, 6t, 2 >$, and $|v(t)| = \sqrt{8t^2 + 9t^4}$.

13. $v(t) = e^t < \cos t, \sin t, t > +e^t < -\sin t, \cos t, 1 >= e^t < \cos t - \sin t, \sin t + \cos t, t + 1 >$. $|v(t)| = e^t\sqrt{(\cos t - \sin t)^2 + (\sin t + \cos t)^2 + (t + 1)^2} = e^t\sqrt{t^2 + 2t + 3}$. $a(t) = e^t < \cos t - \sin t, \sin t + \cos t, t + 1 > +e^t < -\sin t - \cos t, \cos t - \sin t, 1 >= e^t < -2\sin t, 2\cos t, t + 2 >$.

15. $a(t) =< 1, 2, 0 >$, so $v(t) =< 1, 2, 0 > t + C$. Since $v(0) =< 0, 0, 1 >$, $C =< 0, 0, 1 >$. Thus $v(t) =< t, 2t, 1 >$. Now we have $r(t) - r(0) = \int_0^t < u, 2u, 1 > du = < u^2/2, u^2, u > \Big|_0^t =< t^2/2, t^2, t >$, so $r(t) =< t^2/2 + 1, t^2, t >$.

19. $r'(t) =< 2t, 5, 2t-16 >$. The speed is $v(t) = |r'(t)| = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256} = \sqrt{8t^2 - 64t + 281}$. We want to minimize this function. $v'(t) = \dfrac{16t - 64}{2\sqrt{8t^2 - 64t + 281}}$, and this is zero for $t = 4$. If $t < 4$, $v'(t) < 0$; if $t > 4$, $v'(t) > 0$. Thus $t = 4$ gives a minimum. The speed then is $v(4) = \sqrt{8(4^2) - 64(4) + 281} = \sqrt{153}$.

27. We use the result of Example 5. We want α such that $800 = \dfrac{150^2 \sin 2\alpha}{9.8}$. We get $\sin 2\alpha = 0.348\overline{4}$. This gives $2\alpha \approx 20.4°$ or $2\alpha \approx 159.6$ Thus $\alpha \approx 10.2°$ or $\alpha \approx 79.8°$.

33. $v(t) =< 3 - 3t^2, 6t >, a(t) =< -6t, 6 >$, so the speed is $\sqrt{(3 - 3t^2)^2 + 36t^2} = 3(t^2 + 1)$. The tangential component is the derivative of this, or $6t$.
If we think of the motion as being in $\mathbb{R}^3$, we have $v(t) =< 3-3t^2, 6t, 0 >$ and $a(t) =< -6t, 6, 0 >$. The normal component is then $\dfrac{| < 3 - 3t^2, 6t, 0 > \times < -6t, 6, 0 > |}{3(t^2 + 1)} = \dfrac{| < 0, 0, 18 - 18t^2 + 36t^2 > |}{3(t^2 + 1)} = 6.$