Solutions to Homework Assignment 12

MATH 249

Section 14.1, Page 897 Stewart 6e

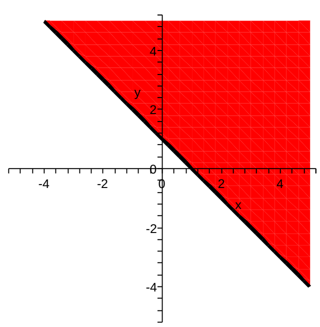
2, 3, 6-13, 19, 20, 21, 24, 25, 28, 29, 30, 31, 32, 39, 40, 45, 55-60

2. (a) $f(95, 70) = 124$. This means that at a temperature of 95 and a humidity of 70%, the apparent temperature is 124. Mercy!
   (b) The humidity that gives an apparent temperature of 100 at an actual temperature of 90 is $h = 60$.
   (c) A temperature of 85 at 50% humidity will yield an apparent temperature of 88.
   (d) These are both functions of one variable. The first gives the apparent temperature as a function of humidity when the actual temperature is 80; the second gives the apparent temperature as a function of humidity when the actual temperature is 100. Both grow with h, but the second one grows much faster.

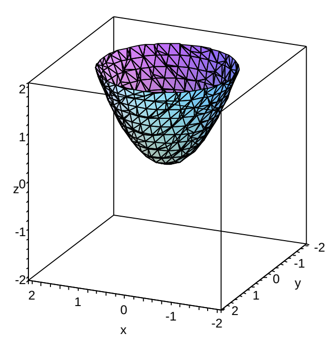
3. $P(2L, 2K) = 1.01(2L)^{0.75}(2K)^{0.25} = 1.01 \cdot 2^{0.75+0.25}L^{0.75}K^{0.25} = 2P(L, K)$. This also works for the general production function since the exponents on L and K sum to 1.

6. (a) $f(1, 1) = \ln(1 + 1 - 1) = 0$.
   (b) $f(e, 1) = \ln(e + 1 - 1) = 1$.
   (c) The domain of $\ln t$ is $(0, \infty)$, so we need $x + y - 1 > 0$, or $y > 1 - x$. The domain is thus the region of the plane above the line $y = 1 - x$, as shown.
   (d) Since $x + y - 1$ will span the entire domain of $\ln t$, the outputs will span the entire range of $\ln t$, which is $(-\infty, \infty)$.

9. (a) $f(2, -1, 6) = e^{\sqrt{6-2^2-(-1)^2}} = e$.
   (b) We only require that $z - x^2 - y^2 \geq 0$, or $z \geq x^2 + y^2$. This means that the points in the domain lie on or above the circular paraboloid $z = x^2 + y^2$.
   (c) Since the range of $\sqrt{z - x^2 - y^2}$ is $[0, \infty)$, the range of f is $[1, \infty)$.



Number 6



Number 9

11. Here we need $x + y \geq 0$, or $y \geq -x$. This is the region in the plane above the line $y = -x$.

12. We need both $x \geq 0$ and $y \geq 0$ or $x \leq 0$ and $y \leq 0$, so this is the first and third quadrants plus the origin and axes.

13. We need $9 - x^2 - 9y^2 > 0$, or $\dfrac{x^2}{9} + y^2 < 1$. This is the inside of the ellipse $\dfrac{x^2}{9} + y^2 = 1$ (excluding the ellipse itself).

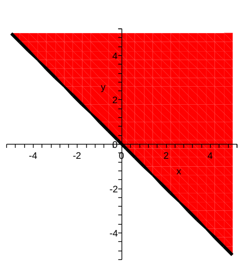
19. We need $x^2 + y^2 + z^2 \leq 1$, so the domain is the unit sphere together with its interior.

21. This is a horizontal plane at height 3.

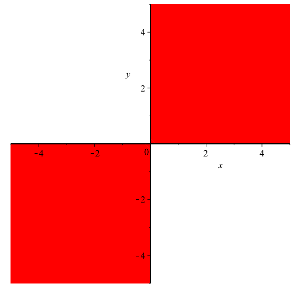
24. This is a sinusoidal cylinder with the y-axis as its axis. Cross sections are $\cos x$.

25. This is a parabolic cylinder with the y-axis as its axis. Cross sections are $1 - x^2$.

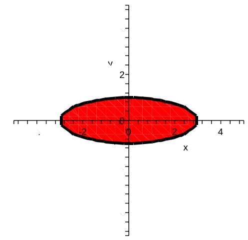
29. This is the top half of a cone; note that $z^2 = x^2 + y^2$ and $z \geq 0$.



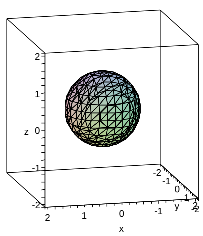
Number 11



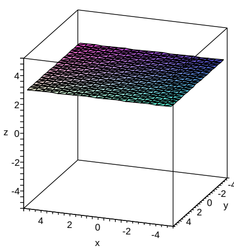
Number 12



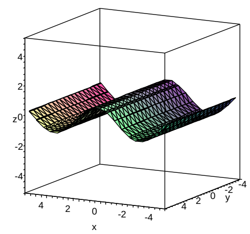
Number 13



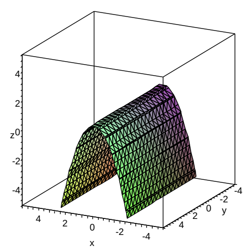
Number 19



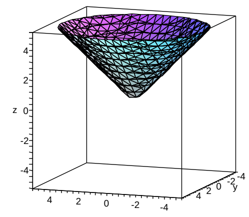
Number 21



Number 24



Number 25

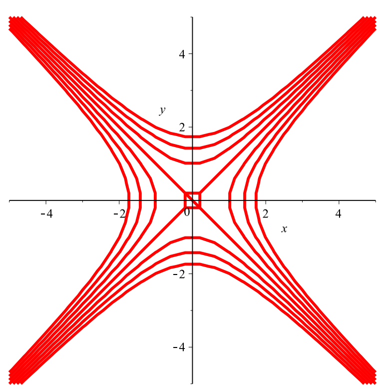


Number 29

31. The graph seems to rise to a point at the origin. $f(-3, 3) \approx 55$ since it is roughly midway between the $z = 50$ and $z = 60$ level curves.

32. There is actually not enough information given, but I will assume that corresponding level curves are at the same height. In I, the drop in the level curves slows down as we approach the origin, so it will have a rounded bottom and therefore be the paraboloid. In II, the dropoff is at a constant rate, so this will be the cone.

45. A contour $k = y^2 - x^2$ is a hyperbola.



Number 45