

Solutions to Homework Assignment 14

MATH 249

Section 14.3, Page 888 Stewart 6e

4, 5, 9, 10, 15, 20, 25, 27, 28, 29, 30, 39, 47, 55, 59, 84, 87, 89

4. (a) $\partial h/\partial t$ gives the rate of change in the wave heights as time changes but wind velocity is held constant. $\partial h/\partial v$ gives the rate of change in the wave heights as wind velocity changes but time is held constant.
- (b) For $f_v(40, 15)$, I will hold t at a constant 15 and find the slopes of secant lines near $v = 40$.
$$\frac{f(50, 15) - f(40, 15)}{50 - 40} = \frac{11}{10} \text{ and } \frac{f(40, 15) - f(30, 15)}{40 - 30} = \frac{9}{10}.$$
Averaging these gives $f_v(40, 15) \approx 1$.
Now holding v at a constant 40, I will find the slopes of secant lines from $t = 15$.
$$\frac{f(40, 15) - f(40, 10)}{15 - 10} = \frac{4}{5},$$
 and
$$\frac{f(40, 20) - f(40, 15)}{20 - 15} = \frac{3}{5}.$$
Averaging these gives $\frac{7}{10}$.
 $f_v(40, 15) \approx 1$ means that we would expect that after 15 hours, if the wind velocity were 41 knots (rather than 40), then the wave heights would be 1 foot higher than they were at 40 knots.
 $f_t(40, 15) \approx 0.7$ means that for winds of 40 knots, we would expect 16 hours to give us wave heights that are 0.7 feet higher than they are after 15 hours.
- (c) Since the wave heights seem to settle down to more or less constant values as we read across rows of the table, it looks like f is becoming nearly constant as time passes. Thus $\lim_{t \rightarrow \infty} \frac{\partial h}{\partial t} = 0$.
5. (a) Note that the positive x -direction is to the left and the positive z -direction is up. If we follow the curve passing through $(1, 2)$ to the left, we see that it is going up, so we expect $f_x(1, 2) > 0$.
- (b) The positive y -direction is toward us. If we follow the curve through $(2, 1)$ coming toward us, it appears to drop down (rapidly, in fact), so $f_y(1, 2) < 0$.
9. Consider the front face of each box. The curves appearing there are in the plane $x = 3$. We are looking for one of the three boxes to be f_y to one of the other two (since x is held constant here). Both (b) and (c) have roughly the same shape on the front face, and neither appears to be the derivative of the other. (Notice that around $y = -2$ both have slopes of zero, but neither has the *value* zero.) Thus (a) is f_y .
Now look along the right face of each box. This corresponds to the plane $y = 3$, so we are now trying to consider f_x . Since (b) has a roughly constant upward slope on this face but (c) is not constant, (c) cannot be f_x if (b) is f . Thus it must be the other way around, so (c) is $f(x, y)$, (b) is $f_x(x, y)$, and (a) is $f_y(x, y)$.
10. The blue numbers indicate the z -values above the corresponding red curves. To estimate $f_x(2, 1)$, we hold y constant at 1 and read the z -values from the graph near $x = 2$. We have $f_x(2, 1) \approx \frac{f(3, 1) - f(2, 1)}{3 - 2} \approx \frac{14 - 10}{1} = 4$. Also, $f_x(2, 1) \approx \frac{f(2, 1) - f(1, 1)}{2 - 1} \approx \frac{10 - 7.5}{1} = 2.5$. Averaging these gives 3.25.
Holding x at a constant 2 now, we get $f_y(2, 1) \approx \frac{f(2, 2) - f(2, 1)}{2 - 1} \approx \frac{8 - 10}{1} = -2$ and $f_y(2, 1) \approx \frac{f(2, 1) - f(2, 0)}{1 - 0} \approx \frac{10 - 12}{1} = -2$. Thus it appears that $f_y(2, 1) \approx -2$.
13. $f_x = 3, f_y = -8y^3$.
15. $z_x = e^{3y}, z_y = 3xe^{3y}$.
25. $f_r = \ln(r^2 + s^2) + r \cdot \frac{2r}{r^2 + s^2} = \ln(r^2 + s^2) + \frac{2r^2}{r^2 + s^2}$. $f_s = \frac{2rs}{r^2 + s^2}$.
27. $u_t = e^{w/t} + t(-w/t^2)e^{w/t} = e^{w/t}(1 - w/t)$. $u_w = t(1/t)e^{w/t} = e^{w/t}$.
28. We use the Second Fundamental Theorem of Calculus: If $F(x) = \int_a^x f(t)dt$ and f is appropriately continuous, then $F'(x) = f(x)$. Thus $f_x(x, y) = \cos(x^2)$. To find f_y , we first rewrite f as $f(x, y) =$

$$-\int_x^y \cos(t^2) dt, \text{ so } f_y(x, y) = -\cos(y^2).$$

39. $f_x(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$, so $f_x(3, 4) = \frac{3}{5}$.

59. $u_x = \frac{1}{\sqrt{x^2 + y^2}} \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{x^2 + y^2}$. $u_y = \frac{y}{x^2 + y^2}$. $u_{xy} = \frac{-2xy}{(x^2 + y^2)^2}$, $u_{yx} = \frac{-2xy}{(x^2 + y^2)^2}$.

87. Since f_x and f_y are both continuous, we must have $f_{xy} = f_{yx}$ by Clairaut's theorem. However, we have $f_{xy} = 4$ and $f_{yx} = 3$, so this is not the case. Someone lied to us.