Solutions to Homework Assignment 15

MATH 249

Section 14.4, Page 899

1, 5, 6, 11-14, 19, 25, 27, 30, 31, 34, 36, 37

1. $z_x = 8x, z_y = -2y + 2$. Thus $z_x(-1, 2) = -8$ and $z_y(-1, 2) = -2$. We get $z = -8(x+1) - 2(y-2) + 4$, or $z = -8x - 2y$.

5. $z_x = -y\sin(x-y)$, so $z_x(2, 2) = 0$. $z_y = \cos(x-y) + y\sin(x-y)$, so $z_y(2, 2) = 1$. We get $z = (y-2)+2$, or $z = y$.

11. $f_x = \sqrt{y}$ and $f_y = \dfrac{x}{2\sqrt{y}}$, both of which are continuous at $(1, 4)$. $L(x, y) = f_x(1, 4)(x-1) + f_y(1, 4)(y-4) + f(1, 4) = 2(x-1) + \dfrac{1}{4}(y-4) + 2 = 2x + \dfrac{1}{4}y - 1$.

19. $f_x = -\dfrac{x}{\sqrt{20 - x^2 - 7y^2}}$, so $f_x(2, 1) = -\dfrac{2}{3}$. $f_y = -\dfrac{7y}{\sqrt{20 - x^2 - 7y^2}}$, so $f_y(2, 1) = -\dfrac{7}{3}$. We get $L(x, y) = -\dfrac{2}{3}(x-2) - \dfrac{7}{3}(y-1) + 3 = -\dfrac{2}{3}x - \dfrac{7}{3}y + \dfrac{20}{3}$. Now $f(1.95, 1.08) \approx L(1.95, 1.08) = -\dfrac{2}{3}(1.95) - \dfrac{7}{3}(1.08) + \dfrac{19}{3} \approx 2.8467$.

25. $dz = 3x^2\ln(y^2)dx + \dfrac{2x^3}{y}dy$.

30. $dw = (ye^{xz} + xyze^{xz})dx + xe^{xz}dy + x^2ye^{xz}dz$.

31. $dz = 10xdx + 2ydy$. Thus $dz = 10(1)(0.05) + 2(2)(0.1) = 0.9$. On the other hand $\Delta z = [5(1.05)^2 + 2.1^2] - [5(1)^2 + 2^2] = 0.9225$. These are pretty close!

34. The surface area function is $S(x, y, z) = 2(xy+xz+yz)$. Thus $dS = 2(y+z)dx + 2(x+z)dy + 2(x+y)dz$. The maximum error is then about $2(60 + 50)(0.2) + 2(80 + 50)(0.2) + 2(80 + 60)(0.2) = 152\text{cm}^2$.

37. This is much like our picture frame example in class. We have $A(x, y) = xy$, so $dA = ydx + xdy$. We get $dA = 200(0.5) + 100(0.5) = 150$ sq. ft. Note that we used $dx = dy = 0.5$ since the 0.25-foot stripe appears on all four sides of the rectangle.